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NOTES ON CALCIUM CLOUDS

By OTTO STRUVE

ABSTRACT

I. The existence of condensations in the interstellar cloud of gas is inferred from a comparison of distances determined by means of the effect of galactic rotation and by means of the intensities of interstellar *Ca* II lines.

II. From the absence of any measurable second-order effect of galactic rotation in the radial velocities determined from interstellar lines, the distance of the galactic center is estimated to be at least 10,000 parsecs.

I. CONDENSATIONS IN THE INTERSTELLAR CALCIUM CLOUD

The mean relative distance of the *Ca* II atoms which contribute to the formation of an interstellar line can be determined either from the effect of galactic rotation upon the residual radial velocity or from the intensity of the line. The two methods should, within the errors of measurement, give concordant results if the calcium atoms are uniformly distributed in space, and if the basic assumptions of galactic rotation are fulfilled. As Eddington has pointed out, "the motion of the cloud must correspond almost exactly to that of a particle revolving in a circular orbit about the centre."[1] There is at present no definite evidence of the existence of individual calcium clouds, or of condensations in the general substratum. The close relation between distance and intensity of interstellar lines[2] suggests that any irregularities in the density of the cloud cannot be very

[1] *The Rotation of the Galaxy: Halley Lecture* (Oxford), p. 28, 1930.

[2] O. Struve, *Monthly Notices of the Royal Astronomical Society*, **89**, 567, 1929.

pronounced.[3] The work of Plaskett and Pearce[4] proves that, statistically, the effect of galactic rotation determined from the interstellar lines is one-half that determined from the stellar lines. However, in discussing the dispersion of their intensity estimates in selected regions, Plaskett and Pearce[5] find it difficult to reconcile their relatively large dispersions with the general uniformity of distribution of interstellar matter shown by their solutions for galactic rotation. Local irregularities in the distribution of the diffuse matter, or variations in density with galactic longitude and latitude, are therefore not impossible. It should be noted, however, that among the hundreds of early B stars observed at the Yerkes Observatory, there is not one which does not possess an interstellar line, provided its distance is sufficiently great. The average number of Ca^+ atoms per cubic centimeter[6] is about 7.5×10^{-8}. The number of atoms required to produce a calcium line near the limit of visibility is about 3×10^{13}. Accordingly, a star would not be expected to show interstellar K unless its distance is greater than about 150 parsecs.

From the work of Strömberg,[7] of Plaskett and Pearce,[8] and of Oort,[9] it is known that the motion of the solar system with regard to the calcium cloud is identical with that determined for the system of the nearer stars. The radial component of the solar motion may thus be computed for any star. Figure 1 shows the residual radial velocities, $R=V-V_0\cos\varphi$, where φ is the angular distance from the apex, for all stars of class B2 or earlier which have *Ca* II intensities of 7 or greater, on the scale used by Plaskett and Pearce. The effect of galactic rotation is well pronounced, but there are several stars in the vicinity of galactic longitude 100° for which the residual velocity is surprisingly small. I have redetermined the radial velocities of three stars. My results, together with those of other investigators, are shown in Table I. The radial velocities of all three stars are known with considerable precision. Incidentally, my measure-

[3] A. S. Eddington, *Observatory*, **52**, 133, 1929.

[4] *Publications of the Dominion Astrophysical Observatory*, **5**, No. 3, 1933.

[5] *Ibid.*, p. 209.

[6] Struve, *op. cit.*, **89**, 584, 1929.

[7] *Astrophysical Journal*, **61**, 372, 1925. [8] *Loc. cit.*

[9] *Bulletin of the Astronomical Institutes of the Netherlands*, **5**, 192, 1930.

ments confirm the conclusion of Plaskett and Pearce,[10] that the variation in the velocity determined by Lee[11] from the *Ca* II lines in 9 α Camelopardalis is probably not real: a plot of my observations shows no trace of the periodicity of eight days announced by Lee.

I have next computed the residual velocity predicted from the intensities of the interstellar lines:[12]

$$\rho = 2.20\,(I - 1.5)\,\sin 2\,(l - 331^\circ)\,\cos^2 b\,.$$

The results for the three stars are given in Table II. The discrepancy is glaring in 9 α Camelopardalis and in 19 Cephei. In both

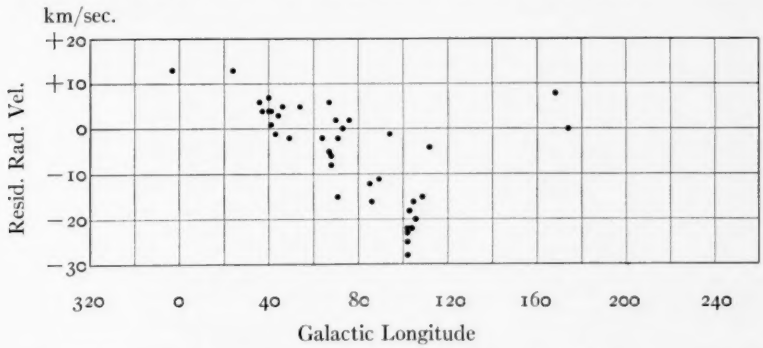


FIG. 1.—Residual radial velocities from interstellar calcium lines of intensity greater than seven on the scale of Plaskett and Pearce.

stars the distance determined from the intensities is far greater than that from the rotational effect; thus, for 9 α Camelopardalis the rotational effect gives

$$D_{Ca\,II} = 300 \text{ parsecs}; \quad D_{star} = 600 \text{ parsecs}\,,$$

while the intensity gives

$$D_{Ca\,II} = 1200 \text{ parsecs}; \quad D_{star} = 2400 \text{ parsecs}\,.$$

The precision of the radial velocities is such that the two sets of distances cannot be reconciled, nor can the error be in the intensities. In order to obtain $D = 300$ parsecs, the intensity of *Ca* K should have been between 3 and 4 instead of 10.3.

[10] *Op. cit.*, p. 236. [11] *Astrophysical Journal*, **37**, 1, 1913.

[12] Plaskett and Pearce, *op. cit.*, p. 197 and p. 199.

TABLE I

9 α CAMELOPARDALIS ($l=112°$; $b=+15°$; Int. = 10.3)

	Vel. Km/Sec.	P.E. Km/Sec.	No. of Plates
Struve	− 7.3	±0.6	15
Pearce	− 7.1	.3	7
Lee	− 5.8	±0.4	79
Adopted mean	− 7.0		
Solar motion	+ 2.4		
Residual velocity	− 4.6		

19 CEPHEI ($l=73°$; $b=+5°$; Int. = 9.0)

	Vel. Km/Sec.	P.E. Km/Sec.	No. of Plates
Struve	−11.3	±0.7	5
Pearce	−12.8	±0.6	16
[Struve	− 5.2		1] omitted
Adopted mean	−12.0		
Solar motion	+13.0		
Residual velocity	+ 1.0		

9 CEPHEI ($l=70°$; $b=+7°$; Int. 5.0)

	Vel. Km/Sec.	P.E. Km/Sec.	No. of Plates
Struve	−21.1	±0.2	3
Struve	−24.1	±1.7	3
[Lee	+ 5.0		2] omitted
Adopted mean	−22.0		
Solar motion	+13.7		
Residual velocity	− 8.3		

TABLE II

Star	ρ	R Obs.	R − ρ
9 α Cam	−17.8	−4.6	+13.2
19 Ceph	− 6.7	+1.0	+ 7.7
9 Ceph	− 2.3	−8.3	− 6.0

There are three possible explanations:

1. That stellar lines of *Ca* II blend with the interstellar lines. This does not apply to 9 α Camelopardalis and to 19 Cephei because both stars are of class O9, so that the ordinary stellar lines should not be observable, and because in both stars the stellar lines are wide and diffuse, so that no effect upon the narrow and strong interstellar line may be expected.

2. That there are condensations in the general cloud. This is the most probable explanation. Whether the condensations are near the stars, as was suspected by R. K. Young[13] and by Lee,[14] or whether they are somewhere along the line of sight, is uncertain. It is probable that the condensations are large enough to affect several stars appearing near one another in the sky. Thus, there are several stars near 19 Cephei which have very strong interstellar lines and consequently lead to positive values of $(R-\rho)$. If it is permissible to assign to a calcium cloud an apparent angular diameter of 5°, we obtain at $D_{Ca\,II}=300$ parsecs a linear diameter of 26 parsecs. The normal number of atoms over a distance of 300 parsecs is 1.7×10^{14}; the observed number is roughly 5×10^{14}. Accordingly, the density within the condensation is

$$\Delta=\frac{3\times10^{14}}{26\times3\times10^{18}}=3\times10^{-6}\ \text{atoms/cm}^3\ .$$

This is about forty times greater than the normal density of the calcium substratum.

3. If the existence of condensations is admitted, we must also admit the possibility of peculiar motions of the clouds, which may vitiate the results obtained from the application of the theory of galactic rotation. This may perhaps explain the positive sign of the residual velocity for 19 Cephei. With a galactic longitude of 73°, the effect of galactic rotation should have been negative.

It is of interest to find that for 9 Cephei, four degrees away from 19 Cephei, the residual velocity is almost normal. It is probable that the intensity of *Ca* K is somewhat greater than 5.0, and that consequently the discrepancy between ρ and R may be less than 6.0 km/sec.

[13] *Publications of the Dominion Astrophysical Observatory*, 1, 219, 1920. [14] *Loc. cit.*

I have attempted to make a statistical study of the calcium clouds by plotting upon a map the values of $(R-\rho)$ computed from the data in the Victoria catalogue. Systematically negative values indicate relatively low intensities, while positive values indicate relatively strong lines. There are several regions which show unmistakable systematic effects. Some of these are listed in Table III. Whether any physical reality should be attributed to these regions is, however, doubtful; the intensities in many stars are quite uncertain, and even the radial velocities are not known with great precision.

TABLE III

Region	α	δ	Sign of (R−p)
Cygnus	19^h40^m to 20^h15^m	+30° to +40°	Negative
Perseus	2 0 to 2 40	55 to 60	Negative
Cepheus	21 30 to 22 10	57 to 62	Positive
Auriga	4 45 to 5 30	+30 to +40	Positive

The regions in Cepheus and in Auriga correspond with regions in which Stebbins and Huffer[15] find strong reddening of the B stars. My negative region near the double cluster of Perseus, however, also shows conspicuous reddening. There is no obvious correlation between the two effects.

II. SECOND-ORDER EFFECTS OF GALACTIC ROTATION

The radial velocities of the interstellar calcium lines, freed from the component of solar motion, reveal no measurable second-order effects of galactic rotation. The theory of these effects has been given by Bottlinger[16] and others. From geometrical considerations it is clear that the effect of galactic rotation should be zero in the following galactic longitudes:[17]

$$l_0;\ l_0 \pm 90^\circ \mp \arcsin \frac{D}{2R};\ l_0 + 180^\circ,$$

[15] *Proceedings of the National Academy of Science*, **19**, 600, 1933.

[16] *Veröffentlichungen der Universitäts-Sternwarte zu Berlin-Babelsberg*, **10**, Heft 2, 1933.

[17] This corrects an error by a factor of 2 in *Zeitschrift für Astrophysik*, **1**, 321, 1930.

where l_0 is the direction of the galactic center. According to Plaskett and Pearce, $l_0 = 331°$. The second-order term is best investigated where the curve of the residual radial velocities plotted against galactic longitudes crosses the zero axis. A plot of the velocities determined from the strongest interstellar lines, corresponding roughly to $D_{Ca} = 1000$ parsecs, gives

$$l_0 + 90° - \text{arc sin } \frac{D}{2R} = 60°,$$

with an uncertainty of about 3°. Hence it would seem that R can hardly be much less than 10,000 parsecs, since we have

R	5000 parsecs	10,000 parsecs	15,000 parsecs	20,000 parsecs
arc sin $\frac{D}{2R}$	6°	3°	2°	1°.5

However, these results depend upon the solution of Plaskett and Pearce for l_0 in which second-order effects were neglected. It is possible that since their material covers only part of the galactic circle, their solution is slightly influenced by these effects. To obtain a separation of the unknowns l_0 and R, it is obviously desirable to cover the entire sky.

If Shapley's center of the galaxy, $l_0 = 325°$, is used, then arc sin $D/2R$ is $-5°$, which is impossible.

It is, of course, possible that the motion of the calcium cloud is not strictly circular, but, according to Eddington,[1] deviations from circular orbits are improbable. At present it seems more probable that the center of the galaxy is at least 10,000 parsecs[18] away and that the observations are not accurate enough to reveal second-order effects amounting at maximum to 3° or less.

YERKES OBSERVATORY
October 1933

[18] *Note added February 2.* This *minimum value* is in accordance with the result of Plaskett and Pearce presented at the Cambridge meeting of the American Astronomical Society in December, 1933. Following the method of Oort they find for the distance of the galactic center approximately 10,000 parsecs.

THE RELATION BETWEEN COLOR EXCESS AND INTERSTELLAR CALCIUM LINE INTENSITY[1]

By E. G. WILLIAMS[2]

ABSTRACT

The spectra of sixty-seven stars have been measured for interstellar K-line intensity on spectrograms giving dispersions of 16.6 and 23.5 A/mm. The dispersion is insufficient to give true line shapes, but the total absorption is obtained with a probable error of about 5 per cent. The spectrophotometric method is described. Comparison with previous intensity estimates shows considerable scatter. The ratio of K to H is 1.61, but a systematic error in the measurement of H in sharp hydrogen-line stars is found to cause this large value. For 12 plates of n-line stars, K/H = 1.43.

The correction to be applied to photo-electric color indices of B- and A-type stars on account of absorption in the Balmer series has been determined. For Elvey's color measures of A stars it is about 0.5 mag. The corrected color excesses from three sources are combined. The average deviation from the mean is 0.017 mag.

A statistical correlation is found between color excess and K intensity. The dispersion indicates that the scattering material and interstellar calcium are not coextensive. The Be stars are too red for their calcium intensity. For the six stars with strongest emission the excess redness on the Madison scale is +0.18 mag. No dependence of color excess on the duplicity of the bright lines can be detected. Although the relation of K intensity to color excess does not change systematically for stars within 26 parsecs of the galactic plane, two stars at greater distances are 0.1 mag. too white for their K intensities. No marked longitude effect is apparent.

An attempt to correlate field richness in the neighborhood of each star with S, the space reddening per unit of K-line intensity, shows that S is 50 per cent greater in the obscured regions than in the rich star clouds.

There has been a growing tendency during the past few years to explain the abnormal yellow colors of numerous early-type stars as an effect of differential light scattering in interstellar space. Trumpler's[3] researches on the galactic open clusters made it clear that we had to deal, not only with a marked general opacity, but also with a selective absorption that would produce a very pronounced reddening of the more distant Milky Way objects. Trumpler concluded that the observed relation between the angular diameters of the clusters and the apparent magnitudes and types of their brightest stars required a photographic absorption of 0.67 mag. per kiloparsec and a visual absorption of about half this amount.

Later work has amply confirmed space reddening. Photo-electric

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 487.

[2] Fellow in Astronomy on the Commonwealth Fund.

[3] *Publications of the Lick Observatory*, 14, 154, 1930.

colors of early-type stars show a statistical increase in redness with decreasing apparent brightness which it is impossible to explain as other than a distance effect. Even more conclusive are the results from color measures of the globular clusters. In general, the closer these objects are to the galactic plane, the redder they are, with the result that there remains small doubt that in or near the galactic belt, at least, their light undergoes marked selective absorption over the large distances involved.

There remain three major questions for settlement: (*a*) Is the distribution of scattering material tolerably uniform? (*b*) Does the non-scattered light retain apparent black-body distribution appropriate to some lower temperature? (*c*) Is there an appreciable effect due to the stars themselves and associated with absolute magnitude or spectral peculiarities?

Several considerations immediately suggest that the distribution of scattering particles is by no means uniform. The presence of faint white stars in Milky Way clouds and globular clusters led Shapley[4] to believe that in general there is no appreciable selective absorption in space. A mean color index of -0.17 was found for nine stars in the Scutum cloud[5] with mean apparent magnitude 14.3. The distance of these stars must be of the order of 10,000 parsecs. Bok[6] has found similar negative colors in the densest parts of the southern Milky Way, although at considerably smaller distances. Evidently these results were deduced from objects seen through exceptionally transparent regions.

Again, the scatter and lack of concentration in both color-excess and apparent-magnitude diagrams for B stars[7] and color-latitude diagrams for globular clusters,[8] and the dependence of color excess on position relative to known dark nebulosities point to large departures from uniformity.

In Table I are the distances and absolute magnitudes of selected

[4] "Star Clusters," *Harvard Observatory Monograph*, No. 2, 1930, p. 118.

[5] *Mt. Wilson Contr.*, No. 133; *Astrophysical Journal*, **46**, 64, 1917.

[6] "A Study of the η Carinae Region," *Harvard Reprint*, No. 77, 1932.

[7] C. T. Elvey, *Astrophysical Journal*, **74**, 309, 1931.

[8] J. Stebbins, *Proceedings of the National Academy of Sciences*, **19**, 222, 1933; A. N. Vyssotsky and E. T. R. Williams, *Astrophysical Journal*, **77**, 301, 1933.

apparently bright stars computed from the color excess with the aid of Trumpler's value for the coefficient on the assumption that the excess is due wholly to uniform selective space absorption. The luminosity deduced for ζ Persei in particular suggests either that this star is peculiar or that space reddening is abnormally strong in its direction. It may fairly be said that at both great and comparatively small distances from the sun the evidence points toward great irregularity in the distribution of scattering material.

TABLE I

Star	m_{vis}	Spectrum	Gal. Lat.	Color Excess (Normal Scale)	Deduced		Edwards' Spectroscopic M_{vis}
					Distance in Parsecs	M_{vis}	
ζ Per	2.9	B1	17°	0.38	1200	−7.1	−3.2
55 Cyg	4.9	B2	1	.64	2000	6.0	2.7
χ^2 Ori	4.9	B2	1	.46	1400	5.3	2.3
67 Oph	3.9	B5	13	0.14:	440:	−4.2	−2.0

On the other hand, considerable confidence may now be felt in the tolerable uniformity throughout the galactic system of the diffuse matter producing the stationary H and K lines of ionized calcium. Confirming Struve's[9] earlier work on the increase of interstellar K-line intensity with distance, Plaskett and Pearce[10] have recently demonstrated in striking fashion that this intensity is exactly proportional to the galactic rotational term for the K line and hence to the distance. They have also shown that for stars with interstellar lines the galactic rotational term is just twice that of the intervening calcium, again indicating the general uniformity of distribution of the latter.

In view of the relatively large spread in the absolute magnitudes of early B-type stars and the present relatively low weight of their spectroscopic parallaxes, the intensity of the interstellar K line is probably as good a criterion of distance as any, provided the intensity can be accurately measured. A study of the relation between space reddening and this intensity will therefore be of assistance in

[9] *Astrophysical Journal*, **67**, 353, 1928.

[10] *Publications of the Dominion Astrophysical Observatory*, **5**, No. 3, 1933.

examining the distribution of the scattering material which produces the color excess.

Existing work on the K-line and color-excess relation has been summarized by Miss Westgate,[11] who found a weak correlation. Her paper was published, however, before the Madison photo-electric colors for all bright B stars were available, and Stebbins and Huffer[12] in discussing these colors remark that "there seems to be no correlation whatever between color-excess and calcium intensity."

OBSERVATIONAL DETAILS

Previous estimates of K-line intensities have suffered to a greater or less degree from the fact that the density of the spectra falls off rapidly toward the ultra-violet. On most ordinary plates the continuous spectrum is decidedly too weak at K for accurate intensity measures.

The spectrograph used in the present investigation was designed primarily for the H and K region, which passes centrally through the optical system and appears on the middle of the plate. The optical train, consisting of three 2-inch light flint prisms and a collimating lens of ultra-violet glass, is exceptionally transparent. Another great advantage is that the spectrograph may be rotated about the axis of the collimator. For each observation the slit was made to coincide with the image of the vertical circle through the star on the slit plate. This operation was facilitated by means of a chart from which the position angle of the spectrograph could be read with the hour angle and declination of the star as arguments. With the instrument thus rotated, the maximum concentration of light of all wave-lengths passes through the slit when one guides on the visible image, an important consideration with a telescope of great focal length. For example, with the 60-inch reflector and a star on the meridian 45° from the zenith, if the visible image is held on a slit parallel to the equator, the image in K light is 0.12 mm from the slit center. With moderately good seeing (3–4 on the Mount Wilson scale) and normal slit width (0.04 mm), approximately 75 per cent of the light at K is then lost. Even with a telescope of 30-foot focal length and a star only 30° from the zenith, the loss would be over 50 per cent.

[11] *Astrophysical Journal*, **78**, 65, 1933.

[12] *Proceedings of the National Academy of Sciences*, **19**, 603, 1933.

For precise photometric work it is of prime importance that the spectra shall be adequately broad and uniform. Normally the width on the plate was 0.7 mm; for the faint stars it was reduced to 0.4 mm. In normal B-type stars the spectra, which extend from λ 5000 to λ 3600, attain maximum density between $H\gamma$ and $H\delta$, and are exceptionally uniform at the K line. Cameras of 15- and 10-inch focal length have been used on the spectrograph. The respective dispersions at K are 16.6 and 23.5 A/mm.

The spectrograph was fitted with a device for standardizing the plates during exposure on the stars. Light from a 15-watt, 110-volt

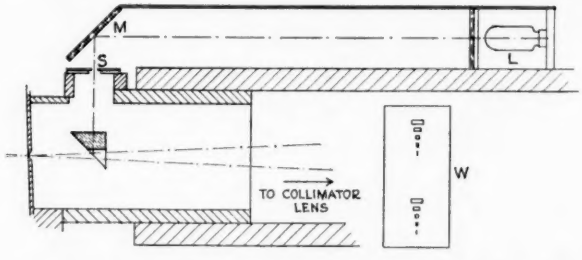


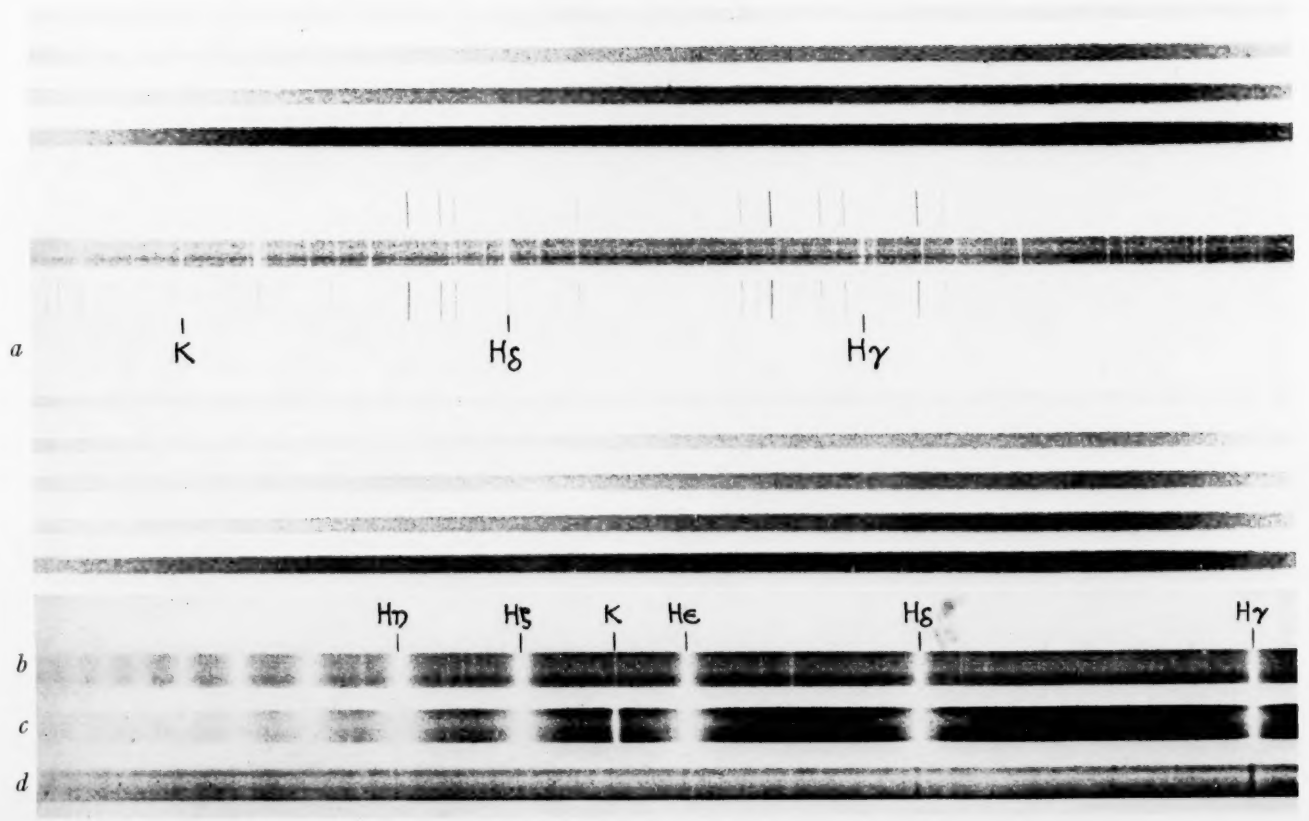
FIG. 1.—Standardizing attachment on the ultra-violet spectrograph

opal-glass lamp (*L*, Fig. 1) falls on a magnesia diffusing surface, *M*, which illuminates uniformly two series of slits of graduated width, accurately cut from copper foil, mounted in front of the long broad aperture *S* (*W*, Fig. 1). The beam enters the spectrograph in a direction perpendicular to the collimator axis. A 90°-prism mounted on the axis reflects the light into the collimating lens, while the star beam from the primary slit passes uninterruptedly through a cut in the central part of the prism. The insides of both collimator and camera were adequately diaphragmed to get rid of stray light, and a blue filter was used to eliminate the excess of green light.

On the plate, the star's spectrum appears flanked by two series each of five continuous spectra of known intensity ratios, with corresponding wave-lengths in stellar and standardizing images in alignment (Pl. IV*a*). Lateral displacement of the plate-holder enabled two or three spectra to be obtained on one plate.

All the spectra were reduced with the aid of the recording photoelectric microphotometer, the magnification from plate to tracing

PLATE IV



(*a*) HD 13854 in Perseus cluster, with standardizing spectra; (*b*), (*c*), (*d*) Spectra for determination of integrated line absorption in the ultra-violet.

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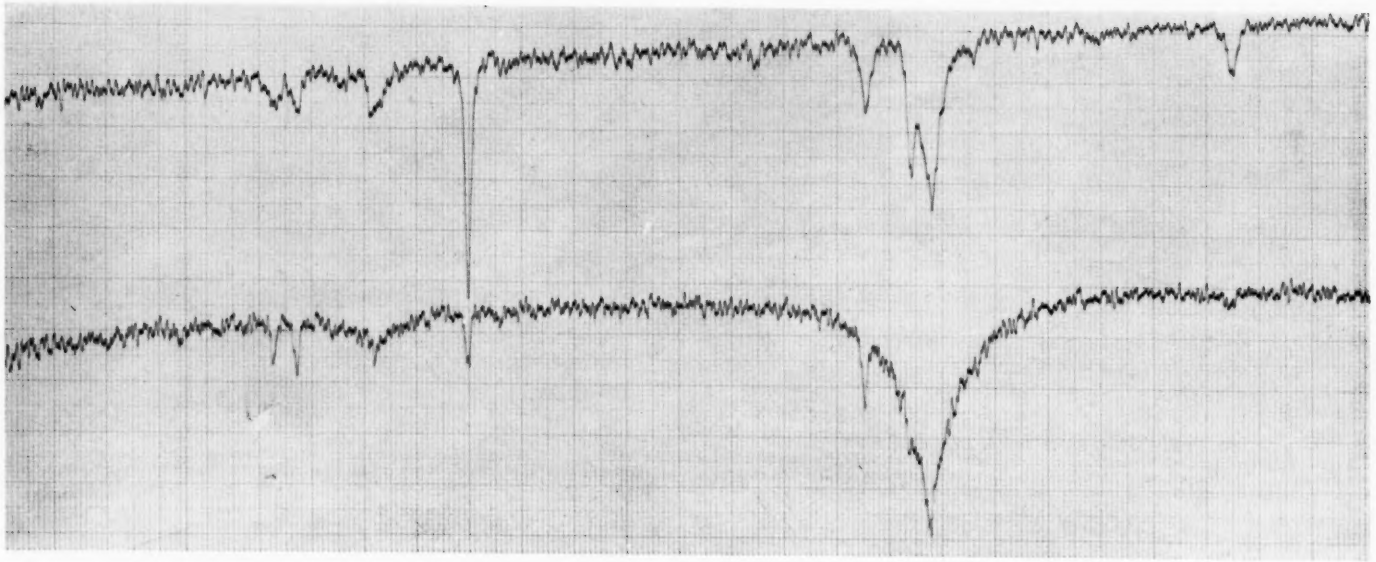
PLATE V



MICROPHOTOMETER TRACINGS OF THE K LINE AND OF *He* AND H IN 55 CYGNI; BELOW, ι HERCULIS

being forty times and the rate of run 4 mm of plate per minute. That this speed allowed time for the galvanometer to follow the line shape was checked by taking much slower runs, which on reduction gave identical results. The virtual width of the analyzing slit at the plate was 0.006 mm. Plate V shows tracings of interstellar lines of widely differing intensity taken with the 15-inch camera.

Photometer readings were reduced to intensities with the aid of tracings of the standardizing spectra obtained by allowing the beam to traverse these spectra in a direction perpendicular to the dispersion. Owing to the curvature of the image of the slit on the plate, such a run measures the density of different wave-lengths at the middle and extremities of the slit image. When the density changes rapidly with wave-length, the error introduced is by no means negligible. To eliminate this error, and also the effect of any departure from uniformity of illumination over the step-slit, a plate was taken with this slit replaced by one of uniform width and standardized by a step-slit exposure developed with it. Measures of this plate, also made perpendicularly to the dispersion, gave the corrective ratios to be applied to the intensities in each standardizing strip at each of five selected wave-lengths.

The actual widths in the step-slit range from 2.97 to 0.08 mm. Neither the slight loss of purity in the broadest strip nor the diffractional loss in the narrowest is of any consequence. For purposes of reduction, log (intensity) was plotted against photometer deflection. For the Imperial "Eclipse" plates used throughout the work, the reduction-curve, fortunately, changed inappreciably with wave-length. The combination of points from five or six different cross-runs to form one curve gave so small a scatter that an error due to faulty calibration of 4 per cent in the measurement of relative intensities of normally exposed images would be unusual.

The plates were backed, and brushed during development. The systematic difference between measurements of spectra taken with the 10- and 15-inch cameras has been found to be negligible.

K-LINE MEASUREMENT

The photometry of the interstellar K line requires especial consideration on account of its unique sharpness. The only measure-

ment of physical significance which can be made with the dispersion used is the total absorption in the line. To demonstrate this we have determined the instrumental redistribution of light which would be produced for two ideal absorption lines comparable in intensity with observed interstellar lines and following Unsöld contours, i.e., having zero central intensity. This was done by measuring spectrophotometrically the apparent intensity distribution on 10-inch camera spectrograms of light in six iron-arc lines, namely, λλ 3895.67, 3899.72, 3920.27, 3922.92, 3927.93, and 3930.31. Arising from low atomic energy levels, these lines are little influenced by pressure effect, and their true breadths in the source used do not exceed 0.09 A. When so exposed as to have central densities comparable with those in the continuous spectra of stars, their average apparent width on the plate is about 1 A. The procedure in computing the change in apparent shape of the absorption lines has been described elsewhere.[13]

In Figure 2, *D* represents the instrumental redistribution of strictly monochromatic light on the same scale as the rest of the diagram. The two adjacent short lines represent the microphotometer and spectrograph slit widths, the latter being the larger. In *A* and *C* the absorption lines reaching zero central intensity represent Unsöld contours, which are modified in the spectrograph to the shallower lines having the same total area. The dots represent observed contours for K lines of roughly the same total absorption (mean contour for ten weak lines in *A*, and for twelve moderately strong lines in *C*). It is seen that even if the faintest lines were completely black at their centers, we should obtain no indication of the fact; but the rough agreement between observed and computed contours does not necessarily imply that such is the case, because the extreme depth of the assumed line has but little influence on the computed instrumental shape.

The difficulties encountered in measurement are of two kinds: (1) accidental errors due chiefly to chance agglomerations of silver grains in the emulsion and (2) systematic errors introduced in drawing the mean shape on the photometer tracing. All plates have been weighted on a scale of 1 to 3 by visual inspection, the criteria being merely

[13] *Annals of the Solar Physics Observatory, Cambridge*, **2**, Part II, 1932.

density, breadth, and uniformity of the spectra at the K line. The average extreme amplitudes of the oscillations on the tracing due to plate grain, expressed as percentage of light intensity in the continuous spectrum at K and at a point where the intensity is half of

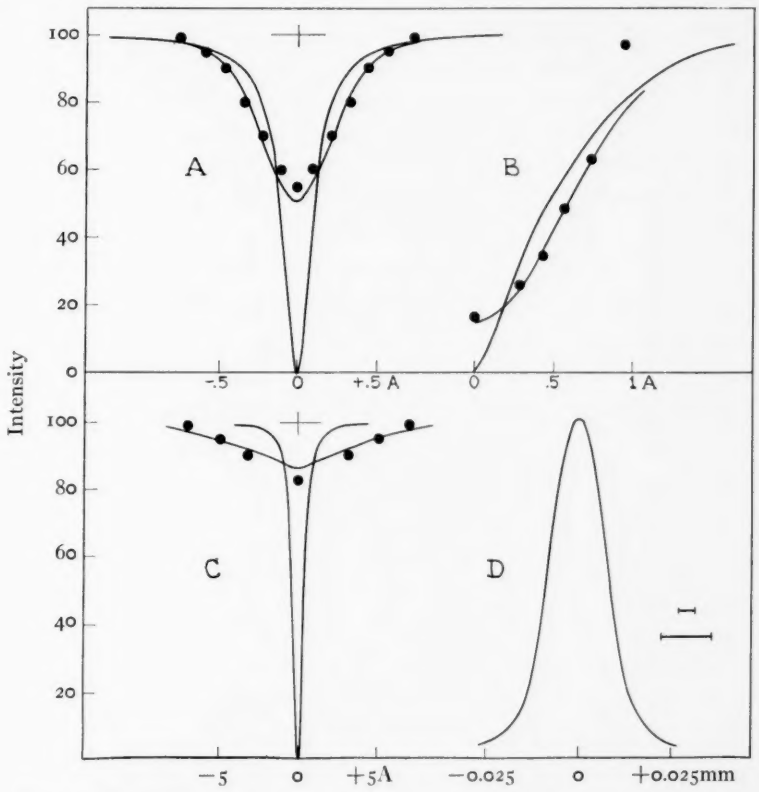


FIG. 2.—Curves showing how the instrumental redistribution of monochromatic light, *D*, alters the shapes of two ideal absorption lines with zero central intensity. *A*, moderately strong K line; *C*, weak line. *B* is a semi-logarithmic plot of the lines in *A*. Dots represent observed contours.

this, are as tabulated. The smoothed continuous spectrum can be drawn on the tracing with an uncertainty of position less than one-fifth of these amplitudes. It is impossible with such grain effects to draw in the wings of faint lines with any accuracy. Figure 2*B* shows the curves in *A* plotted on a semi-logarithmic scale. In the wings the observed points deviate from both the computed instrumental con-

tour and the exponential shape, which on this form of plot is represented by a straight line.

Plate weight	1	2	3
Amplitude in continuous spectrum at K	18	10	6
Amplitude at intensity 50 per cent	40	14	8

To investigate the wings more closely, two spectra of ζ Tauri, B3, were obtained on Eastman Contrast-Bromide plates. Four spectra showing the interstellar line on Eastman 33 and Process plates, taken with the 9-foot coudé spectrograph, were also available. Here again the observed shapes are wholly instrumental, but as the grain effects are minimized, the extreme wings could be measured with some accuracy and on all six plates were found to follow exponential curves fitted to the more central parts of the line. It is probable that a truer figure for the total absorption in K is obtained on the assumption of exponential wings for all lines.

TABLE II

Group	No. of Plates	Mean Measured Total Width	Exponential Width at $r=0.99$	Measured Area	Percentage Increase Applied
I	7	2.21 A	2.50 A	32.5	1
II	15	1.83	2.29	19.0	5
III	18	1.64	2.50	12.2	9
IV	13	1.66	2.39	6.8	15

The conversion to this "exponential total absorption" was made as follows: All the K lines were divided into four intensity groups. For each group the mean measured contour was found by adding the widths at $r=1.0$, 0.95, 0.9, etc., and dividing by the number of stars in the group, r being the ratio of intensity at a point in the line to that at a point just outside it. The mean contours were plotted semi-logarithmically. Straight lines drawn through the more reliably determined parts of the line gave the widths at $r=0.99$ to be used in computing the corresponding exponential total absorptions. The corrections to be applied to the measured areas for each line were found from a curve constructed from the data of Table II. For the weakest measured lines the extreme widths were assumed equal to those found for the next intensity group.

It was soon noticed that measured line depth, here expressed as

percentage absorption of the continuous spectrum, gave more consistent results than total absorption, especially for the weaker lines. Sometimes different spectra of the same star show disconcerting differences in total width. Since a depth determination from a single plate is, however, subject to the foregoing percentage-intensity uncertainty, and since any slight error in focusing or a temperature shift during exposure will affect the depth but not the area, it is unwise to neglect the latter. Further, the calcium intensity derived

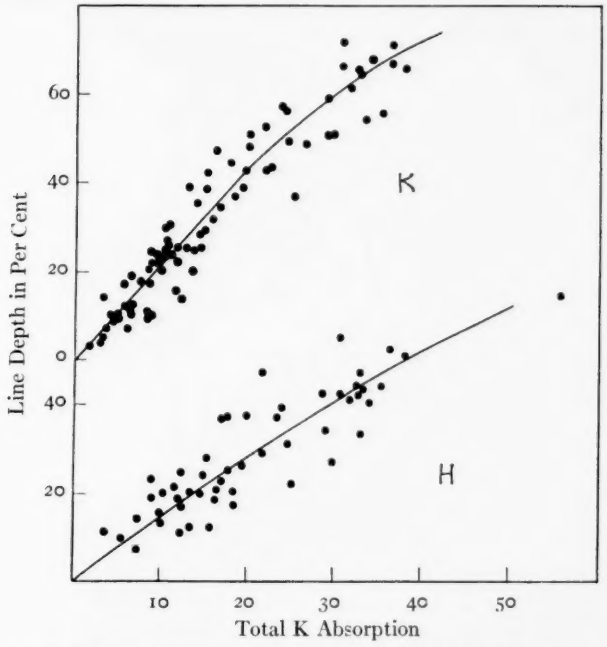


FIG. 3.—The relation between total absorption in K and the depth of K and H

from a single plate should be made to depend on H as well as on K when both lines are subject to considerable accidental error.

The total absorption in the H line is particularly hard to measure, unless the line is strong and $H\epsilon$ weak. The chief difficulty is in outlining the wings on a curved background. The depth expressed as a percentage of the intensity in $H\epsilon$ at the wave-length of H can, however, usually be obtained with some confidence if the line is not too weak.

In Figure 3, the depths of K, above, and H, below, are plotted

against the exponential total absorption in K. The curves have been used to convert observed depths into corresponding areas for combination into a mean total absorption for each plate. For depths less than 50, the area derived from the depth of K has been given double weight and those from measured area and H depth each single weight. The K depth was not used for the two stars with strongest lines; instead, the value from the measured K area was given double weight.

The results from ninety-nine spectra of sixty-seven stars are recorded in Table III. The list comprises early B-type stars, most of which were selected for observation on account of strong or weak K lines, abnormal redness or whiteness, high galactic latitude, or location in dense star clouds. The presence of stellar K lines in classes $B3$ and $B5$ has a negligible effect on the results, though in further work their strength, particularly in class $B5$, should be determined and allowed for. K is weak or invisible in all but one of the $B5$ spectra here measured. The total absorptions are expressed in units of complete absorption per 0.1 wave number. The results in the fourth and fifth columns are directly measured values, uncorrected for exponential wings. Plate numbers in italics refer to 9-foot coudé spectra.

For twelve stars both H and K were measured on more than one plate. Table IV gives the average deviation from mean values in absorption units, and the percentage probable errors from twenty-nine plates. For all stars with total absorptions obtained from two or more plates, the mean residual is 1.06 units (forty-nine plates) and the percentage probable error 6.3 per cent, or 5 per cent if one plate is omitted.

It is surprising to find that the present measures of K-line intensity deviate considerably from the estimates of Plaskett and Pearce[14] made by visual comparison with a series of artificial absorption-line scales. For such a line as K, which does not vary in width, considerable accuracy would be expected by this method from well-exposed spectra.

In Figure 4, which exhibits the relation between the two scales, the stars fainter than magnitude 6.5 are indicated by dots. For a

[14] *Loc. cit.*

TABLE III

MEASUREMENTS OF LINE ABSORPTIONS

STAR	PL.	WT.	TOTAL ABSORPTION		DEPTH		REDUCED K ABSORPTION
			K	H	K	H	
γ Peg	V 140	2	5.6	.....	19	.....	8.0
	289	3	7.0	.....	21	.....	9.1
	342	.....	7.0	.....	17	.....	.....
	345	.....	5.4	.....	13	.....	.....
ζ Cas	261	2	8.7	6.4	23	13	9.9
ο Cas	140	1	6.2	.....	11	.....	6.0
	293	3	7.8	.....	10	.....	6.4
γ Cas	301	3	1.2	.....	3	.....	1.7
	318	3	2.3	.....	4	.....	2.5
ε Cas	302	2	7.9	.....	16	.....	8.2
HD 13854	412	3	50.5	25.9	80	47	45.6
HD 14134	417	2	49.5	29.5	84	49	45.3
δ Per	319	3	7.2	.....	7	.....	5.2
23 Tau	160	2	Invis.	.....	.....	.....	.....
ζ Per	141	2	14.2	4.6	25	12	11.6
	259	2	11.1	2.8	24	11	10.4
	285	3	7.5	2.9	24	19	11.0
X Per	286	2	12.6	.....	27	.....	12.7
	397	2	12.2	.....	33	.....	14.3
ξ Per	285	3	7.6	6.4	32	23	13.3
1 Cam Br	303	2	12.2	12.0	33	22	14.6
	329	2	15.6	7.1	26	22	18.0
Ft	303	1	18.1	.....	32	20	15.3
	329	1	14.2	7.3	31	25	15.5
9 (a) Cam	294	3	23.5	13.8	49	31	23.2
	319	3	22.8	15.2	56	39	26.6
11 Cam	415	2	V.ft.	.....	.....	.....	.....
η Aur	141	2	11.0	.....	14	.....	8.5
	152	2	11.3	.....	13	.....	8.4
χ Aur	152	1	36.5	12.5	.....	.....	36.5
	258	3	31.1	23.5	61	41	30.7
λ Ori Br	303	3	15.0	8.0	31	18	14.1
	304	2	11.5	6.0	29	17	12.6
Ft	304	1	17.5	8.5	36	17	15.6
ε Ori	258	3	12.3	.....	20	.....	10.7
ζ Tau	263	3	11.0	7.1	25	19	11.9
	286	1	14.6	.....	.....	.....	15.9
	302	3	15.2	2.6	19	9	9.9
κ Ori	315	3	11.8	.....	14	9	7.9
57 Ori	303	2	16.5	.....	39	.....	17.5
139 Tau	128	2	15.8	11.1	42	24	18.0
χ^2 Ori	79	2	34.1	.....	66	55	37.0
	263	3	33.0	16.8	65	43	32.8
β C Ma	315	3	7.2	.....	11	8	5.9
15 S Mon	129	2	15.3	7.9	39	20	16.5
	307	3	15.4	7.8	39	22	16.8
HD 58050	330	2	15.7	.....	20	17	11.9
η Hya	145	1	Invis.	.....	.....	.....	.....
	338	3	3.2	.....	5	.....	3.2

TABLE III—*Continued*

Star	Pl.	Wt.	Total Absorption		Depth		Reduced K Absorption
			K	H	K	H	
23 Sex	308	1	5.6	.....	18	.....	7.7
ρ Leo	145	1	17.6	6.9	34	27	17.1
	167	1	10.0	5.1	20	11	9.2
	297	3	15.0	4.6	22	11	10.8
HD 93521	305	2	15.8	10.6	29	22	14.6
90 Leo Br	180	2	6.8	.....	11	.....	6.2
κ Dra	162	2	Invis.	.....	.....	.....	.....
	167	1	V.ft.	.....	.....	.....	.....
η U Ma	305	3	Invis.	.....	.....	.....	.....
τ Her	146	2	8.7	.....	17	.....	8.7
χ Oph	348	2	5.3	1.2:	17	10	7.1
τ Sco	396	3	V.ft.	.....	.....	.....	.....
ζ Oph	159	2	5.1	.....	7	.....	4.4
	348	2	3.0	.....	14	.....	5.6
HD 149881	393	3	21.0	12.1	48	26	21.3
ι Her	332	3	11.0	.....	25	.....	11.7
67 Oph	187	2	25.5	11.7	48	27	22.3
	326	2	25.4	10.8	43	21	19.9
HD 167971	396	1	40.7	26.2	80	47	42.3
HD 169454	387	1	36.5	17.3	64	47	34.8
12 Vul	188	2	.....	.....	5	.....	2.8
HD 190603	384	3	33.2	15.2	51	27	25.6
HD 191201	417	3	29.6	18.3	60	44	30.8
HD 192422	383	2	39.5	30.6	55	44	32.9
P Cyg	337	3	20.2	.....	51	.....	22.6
	356	3	23.3	.....	55	.....	25.2
HD 193322	411	3	35.1	20.9	67	43	34.0
HD 194279	410	2	54.0	34.5	79	61	50.4
HD 194839	387	1	42.4	23.7	100:	51	44.5
HD 195592	394	2	32.1	.....	51	34	27.0
55 Cyg	77	2	36.4	20.4	65	44	34.1
	289	3	29.5	16.0	61	38	29.4
59 f^1 Cyg	127	1	7.6	.....	22	7	8.2
	340	2	3.4	2.5	7	11	4.6
69 Cyg	383	2	28.1	16.2	37	22	19.4
β Cep	188	1	7.1	.....	9	.....	5.8
	260	2	5.6	.....	10	.....	5.3
9 Cep	260	2	33.0	12.9	54	33	27.6
HD 208185	419	2	26.5	13.6	46	28	22.5
HD 208392	397	2	26.4	26.0	58	52	30.9
HD 208947	306	2	22.2	6.7	41	22	19.0
	353	2	15.0	9.7	43	26	18.5
HD 210809	418	2	42.9	29.7	70	49	39.2
λ Cep	349	3	18.4	13.3	47	37	22.6
HD 212455	411	2	83.5	52.3	85:	76	80.0
π Aqr	397	3	6.4	4.7	18	14	8.4
6 Lac	306	2	19.0	4.3	35	13	15.0
10 Lac	298	2	7.8	.....	18	.....	8.5
	395	3	9.8	7.1	21	15	10.1
HD 224151	395	3	23.5	15.0	43	29	21.2

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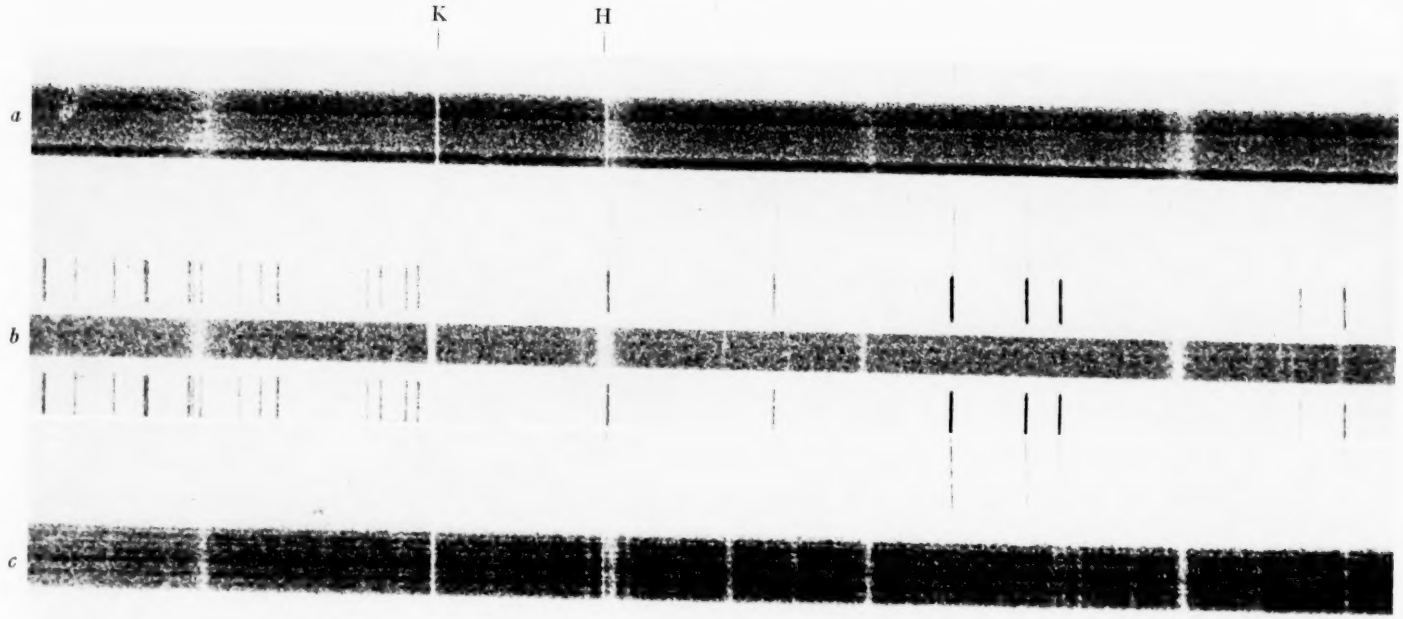
PLATE VI



THE INTERSTELLAR H AND K LINES

(*a*) HD 193322; (*b*) HD 212455; (*c*) HD 190603

given intensity on our scale the Victoria estimates are systematically smaller for the fainter stars. Such an effect would be expected if, on Victoria plates, the density of the continuous spectrum was, for the fainter stars, less than that in the scale with which they were

TABLE IV

ERRORS IN ABSORPTION MEASUREMENT

	Total Absorption From			Weighted Mean
	Measured Area	Depth of K	Depth of H	
Mean residual	1.72	1.18	2.29	1.25
Percentage P.E.	9.7	8.6	14.1	7.1

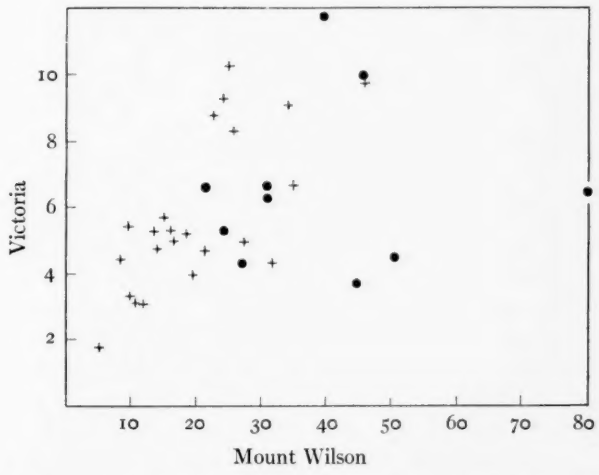


FIG. 4.—Comparison of K line intensities with Plaskett and Pearce's estimates. Dots represent stars fainter than $6^{m}5$.

matched, as the contrast decreases rapidly near the plate threshold. It is impossible to say whether or not the correlation is linear.

The star on the extreme right of the figure, HD 212455, was assigned K intensity 15 by Struve,[15] and there is no doubt that in this star the line is much stronger than in any other star in the present list (Pl. VI*b*).

[15] *Mt. Wilson Contr.*, No. 331; *Astrophysical Journal*, **65**, 163, 1927.

Comparison with previously published spectrophotometric measures can be made for only two stars and is as here shown. The agreement with Beals's values,[16] based on five spectra of each star, is very satisfactory.

Star	Yerkes*		Victoria	Mt. Wilson	
	K	H	K	K	H
9(α) Camelopardalis	30.4	23.5	24.4	24.9	13.3
ξ Persei	.....	.....	13.7	13.3	.....

* Unsöld, Struve, and Elvey, *Zeitschrift für Astrophysik*, **1**, 314, 1930.

THE RATIO OF K AND H

For all stars in which both lines were measured, the weighted mean ratio of the total absorptions in K and H, as obtained directly from the tracings, is 1.83 ± 0.38. This large value is accounted for by the impossibility of measuring the full width of H when superposed on a strong $H\epsilon$.

It has been mentioned that the depth of H is much more reliable than its area. As both H and K have instrumentally produced contours, we may use the depth-area curve for K to find the areas of *both* lines from their measured depths. This procedure results in considerable improvement and yields for the ratio 1.61 ± 0.18.

The difference between this value and the theoretical 1.44 (units proportional to wave-number) is disconcerting. It would seem that the outline of $H\epsilon$ over H has been drawn in systematically too low. Such an error would be more serious for stars in which the contour of $H\epsilon$ is steep. Consequently, the diffuse H-line stars were separated from the sharp and the ratio derived for both groups independently, with results as follows:

31 plates of s-line stars K/H = 1.67 ,
14 plates of n-line stars K/H = 1.47 or 1.43 ,
if 2 plates of low weight are omitted.

There seems no doubt that the discrepancy can be adequately accounted for in this manner. This apparent dependence of the ratio on the character of the stellar spectrum makes it difficult to deter-

[16] *Monthly Notices of the Royal Astronomical Society*, **93**, 585, 1933.

mine with certainty whether there is a change in the ratio with increasing intensity. Such a change was suspected and sought for by Unsöld, Struve, and Elvey.[17] We find

For 21 stars $K < 18$ mean $K/H = 1.55$,
For 24 stars $K \gtrsim 18$ mean $K/H = 1.65$.

The ratio for the second group is higher because it includes a somewhat greater proportion of s-line stars, and we can only say that if such an effect is present, it is not large. The star HD 208392, B3ne with moderately strong K, is worth noting, since from a good plate we find $K/H = 1.16$, a low value which appears to be real. Beals

TABLE V

Observer	Filter 1			Filter 2		
	λe	Short Limit	Long Limit	λe	Short Limit	Long Limit
Elvey*	5100	4600	5900	3850	3650	4200
Elvey and Mehlin†	4750	4250	5250	4250	3750	4750
Stebbins and Huffer‡	4570	4100	5400	4220	3100	4800

* *Astrophysical Journal*, **74**, 298, 1931.
† *Ibid.*, **75**, 354, 1932.
‡ *Proceedings of the National Academy of Sciences*, **19**, 603, 1933.

(*Monthly Notices*, **92**, 207, 1932) from three plates of 9 Camelopardalis obtains 1.29 for the ratio, in exact agreement with the Yerkes value, whereas the present measures give 1.87 for the star.

CORRECTION FOR HYDROGEN ABSORPTION

The determination of color indices involves the measurement of the relative light intensity in two more or less extended ranges of wave-length. If either or both of these ranges include absorption lines or bands produced in the star's atmosphere, the resultant light loss must be allowed for before interstellar absorption can be studied.

In photo-electric work the two ranges are usually rather sharply limited in wave-length by suitable filters. The effective and limiting wave-lengths of the filter and cell combinations of the investigators whose results have been used in the present paper are approximately as in Table V.

[17] *Loc. cit.*

The short wave-length in Elvey's first list falls in the region of strong hydrogen-line absorption at the confluence of higher members of the Balmer series.

In order to correct for this stellar absorption we have compared the distribution of energy in the ordinary photographic region in the spectra of five pairs of stars, one of which in each pair had relatively very weak H lines. As usual, the spectrograph was so rotated that the atmospheric-dispersion spectrum always ran along the slit, which thus admitted equal fractions of the light of each wave-length. This procedure is essential in comparing intensities in the continuous spectra. The two stars were observed at nearly equal zenith distances, one directly after the other on the same plate. (See Pl. IV.) On the resulting microphotometer tracing, the hypothetical continuous background was drawn in over the lines $H\beta$, $H\gamma$, $H\delta$, and the magnitude difference between the two spectra was measured at several wave-lengths in the region of these lines. The plot of these differences against $1/\lambda$ defines a straight line, giving the "gradient" between the two stars.[18]

The prolongation of this straight line toward shorter wave-lengths determined the magnitude differences in the region where the hypothetical continuous background was altogether unobservable for one of the pair of stars. On the assumption that both stars radiated as black bodies, the unobserved background could then be drawn in on the tracing. It is possible that on account of their peculiar spectra the comparison stars used do not themselves radiate as black bodies. It is probable, however, that over the moderately short length of spectrum utilized they do so closely enough for the present purpose. The magnitude loss at any wave-length due to hydrogen absorption was then easily found. Figure 5 shows the approximate sensitivity-curve for the combination atmosphere-objective-filter-photo-cell for light from a B-type star for Elvey's violet filter B_2. The curve, although admittedly rough, is sufficiently accurate for the present purpose. When the observed magnitude losses are applied to it, we obtain the residual intensity distribution in the stellar spectrum as shown under it. The upper diagram is for the A-type star γ Ceti, the lower for γ Pegasi, B2. The broken curve represents

[18] For definition of this term see *Color Temperatures of Stars*, Greenwich, 1932.

an alternative, improbable transmission for the filter. A planimeter was used to find the integrated light loss. The results for the five pairs are in Table VI. The corrections in Table VI imply a magnitude loss under the ultra-violet filter which is 2.3 times that expected

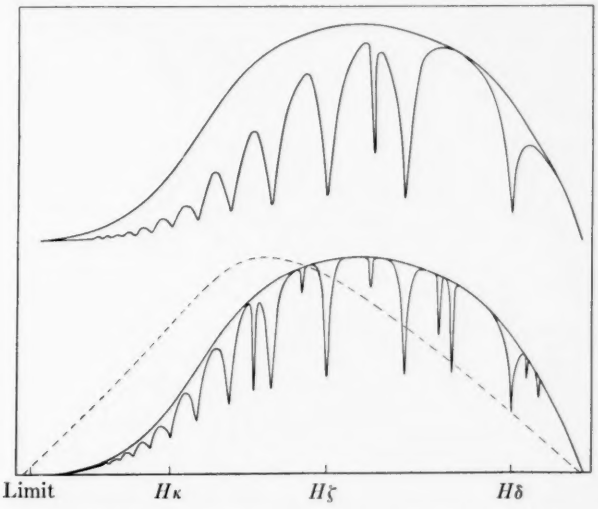


FIG. 5.—Hydrogen absorption under Elvey's ultra-violet filter for γ Ceti, A2 (above) and γ Pegasi, B2 (below).

TABLE VI

Star	Spectrum	Comparison Star	Magnitude Correction	Total Absorption in Hγ	Magnitude Correction for Alternative Transmission-Curve
γ Pegasi	B2	55 Cygni	0.13	235	0.18
ζ Tauri	B3	X Persei	.07	114	.10
β Tauri	B8	γ Cassiopeiae	.18	400	.25
α Coronae	A0	ζ Ophiuchi, β Cephei	.30	780	.42
γ Ceti	A2	γ Cassiopeiae	0.44	1022	0.64

from Elvey's curve[19] correlating color excess with total absorption in $H\gamma$ for his stars.

Since the determination of the hydrogen absorption correction for each star on the program would have involved an unnecessarily

[19] *Op. cit.*, p. 310.

arduous amount of work, the observed losses in Table V were correlated with the total absorption in $H\gamma$, which is measured with relative ease. From the resulting curve in Figure 6 were obtained the corrections applied to the color excess for each star.

For none of the early B stars does the hydrogen absorption correction for the Yerkes green filter $Y2$ used by Elvey exceed 0.01 mag. Similar measures made for the filters used by Stebbins and Huffer gave for γ Ceti:

Photo-cell 302+Blue filter 68*a*, loss 0.23 mag.
Photo-cell 302+Green filter GG5, loss 0.03 mag.

The corrections to the color excesses by the Madison observers were accordingly found by applying the factor 0.45 to the magnitudes read from the curve in Figure 6, and the same factor was found to be applicable to stars in Elvey and Mehlin's list. In neither case do emission lines have any appreciable effect on the measured colors. For γ Cassiopeiae, with strong bright $H\beta$ and $H\gamma$, the net effect on the green magnitudes is less than 0.01.

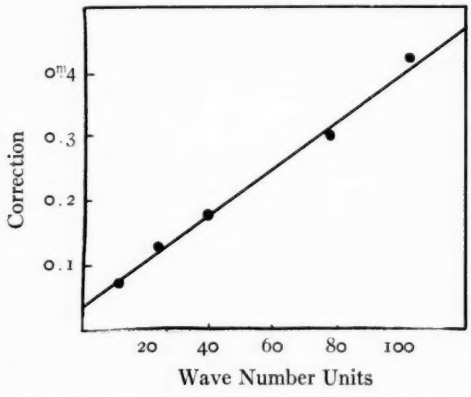


FIG. 6.—Ultra-violet magnitude correction as a function of total absorption in $H\gamma$.

RESULTS AND DISCUSSION

Table VII gathers together the significant material. The spectral types of the bright-line stars are from *Mt. Wilson Contr.* No. 471;[20] for the others, the Victoria classification is given when available. All color excesses are expressed on the Madison scale in units of 0.01 mag., the relation $M = 0.45Y - 0.07$ being used for converting Elvey's corrected color excesses (Y) to this scale (M). Elvey and Mehlin's colors were first converted to Elvey's scale. The color excesses were then found by subtracting the normal color for the subtype, and converted to the Madison scale by the foregoing relation after the

[20] Merrill and Burwell, *Astrophysical Journal*, **78**, 87, 1933.

hydrogen corrections had been applied. Column 11 gives the mean color excess, denoted by E. The average deviation from the mean is ± 0.0167 mag. on the Madison scale.

In Figure 7 the corrected mean color excess of column 11 is plotted against the K-line total absorption, column 7, which is the weighted mean value from Table III. A correlation is at once evident, but there is much scatter, which is certainly not even largely the result of observational errors. In this connection it should be emphasized

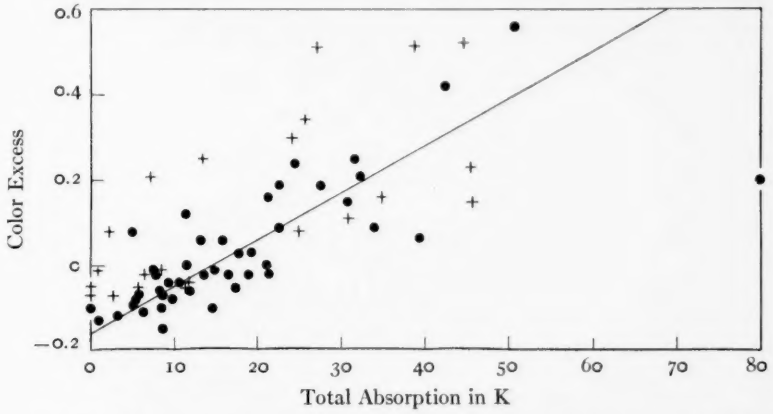


FIG. 7.—Color excess (ordinates) and total absorption in the interstellar K line. The crosses represent bright-line stars.

that the stars represented are by no means typical of the sky as a whole. The majority were chosen on account of extreme color or other peculiarities, and a more general selection would probably reduce the scatter considerably.

As a basis for further discussion, the straight line in Figure 7 has been taken to represent the mean relation between calcium intensity and color excess. The deviations from it, expressed as residual color excess, hereafter denoted by R, are tabulated in Table VII, column 12. For present purposes we shall treat calcium intensity as an index of distance.

THE BRIGHT-LINE STARS

The first thing to be noted in Figure 7 is that the Be stars, represented by crosses, are too red for their distance. In fact, the high percentage of such stars included is due to the selection for study of

TABLE VII

COLLECTED RESULTS

(Unit for color, 0.01 mag.; for log N, 0.01)

HD No.	Star	Mag.	Type	Galactic		Int.	Corrected Color Excess, E				R	log $\frac{N_c}{N_s}$
				Long.	Lat.	K	Y_1	Y_2	M	Mean		
1	2	3	4	5	6	7	8	9	10	11	12	13
886	γ Peg	2.9	B2	78	−47	8.1	−10	− 7	− 5	− 7	− 1	.....
3330	ζ Cas	3.7	B2s	88	−10	9.9	− 7	− 9	− 8	− 8	− 3	+ 2
4180	o Cas	4.7	B4ne	90	−15	6.3	− 1	....	− 3	− 2	+ 7	.....
5394	γ Cas	2.3	Bone	91	− 3	2.1	+ 6	....	+10	+ 8	+20	.....
11415	ϵ Cas	3.4	B3	98	+ 1	8.2	− 5	....	− 7	− 6	+ 1	.....
13854	.....	6.4	B1se	102	− 3	45.6	.....	.....	+15	+15	−19	+12
14134	.....	6.7	B3se	102	− 3	45.3	.....	.....	+23	+23	−11	+13
22928*	δ Per	3.1	B5	118	− 6	5.2	− 9	− 7	−11	− 9	− 3	.....
23480	23 Tau	4.3	B5ne	135	−24	Invis.	− 5	....	− 5	− 5	+12	.....
24398	ζ Per	2.9	B1s	130	−17	11.0	+10	....	+14	+12	+15	−10
24534	X Per	6.2 to 6.9	Bone	131	−17	13.5	.....	.....	+25	+25	+26	.....
24912	ξ Per	4.1	O7n	129	−13	13.3	+ 6	....	+ 6	+ 6	+ 7	+ 5
28446	1 Cam	5.4	B2n	119	+ 4	16.0	.....	.....	+ 6	+ 6	+ 4	− 4
30614	9 (α) Cam	4.4	O9se	112	+14	24.9	+ 8	....	+ 7	+ 8	− 3	+ 1
32343	11 Cam	5.3	B3e	118	+12	V.ft.	− 1	....	− 1	− 1	+14	+ 1
32630	η Aur	3.3	B3	133	0	8.5	− 8	....	−12	−10	− 4	.....
36371	χ Aur	4.9	B1s	144	− 1	32.2	+25	....	+16	+21	+ 2	+ 3
36861	λ Ori	3.7	O8s	163	−12	13.9	− 4	....	− 1	− 2	− 1	− 3
37128	ϵ Ori	1.8	Bo	173	−17	10.7	− 8	....	0	− 4	+ 1	+32
37202	ζ Tau	3.0	B3e	154	− 6	11.6	− 6	....	− 4	− 5	− 2	.....
38771	κ Ori	2.2	Bo	182	−17	7.9	− 6	....	+ 1	− 2	+ 5	+ 7
39698	57 Ori	5.9	B2	157	− 3	17.5	.....	.....	− 5	− 5	− 8	+ 7
40111	139 Tau	4.9	B2	152	+ 1	18.0	.....	.....	+ 3	+ 3	− 1	− 2
41117	χ^2 Ori	4.7	B1se	157	− 1	34.9	+14	....	+17	+16	− 6	+ 6
44743	β C Ma	2.0	B1	194	−13	5.9	− 7	.....	.....	− 7	+ 2	+26
47839	15 S Mon	4.7	O7s	171	+ 3	16.7	0	....	− 5	− 2	− 4	− 6
58050	.....	6.4	B3e	170	+15	11.9	.....	.....	− 4	− 4	− 1	+10
74280	η Hya	4.3	B3	190	+27	3.2	.....	.....	−12	−12	0	.....
89688	23 Sex	6.5	B3	208	+47	7.7	.....	.....	− 1	− 1	+ 6	.....
91316	ρ Leo	3.9	Bos	204	+54	11.7	+ 1	....	− 1	0	+ 3	.....
93521	.....	6.9	B3	150	+63	14.6	.....	.....	−10	−10	−10	.....
100600	90 Leo	5.8	B3	204	+70	6.2	.....	.....	−11	−11	− 2	.....
109387	κ Dra	3.9	B5e	92	+47	Invis.	− 7	....	− 8	− 7	+10	.....
120315	η U Ma	1.9	B3	68	+65	Invis.	− 7	....	−13	−10	+ 6	.....
147394	τ Her	3.9	B5	40	+44	8.7	−15	....	−16	−15	− 8	.....
148184	χ Oph	4.9	B3e	326	+19	7.1	+21	.....	.....	+21	+28	.....
149439	τ Sco	2.9	Bo	319	+11	V.ft.	−13	.....	.....	−13	+ 2	.....
149757	ζ Oph	2.7	Bonn	334	+23	5.0	.....	.....	+ 8	+ 8	+18	.....
149881	.....	6.6	B2	359	+35	21.4	.....	.....	− 2	− 2	− 9	+ 5
160762	ι Her	3.8	B3	40	+30	11.7	− 5	....	− 8	− 6	− 3	.....
164353	67 Oph	3.9	B5s	358	+13	21.1	+ 3	....	− 3	0	− 7	+11

*HD 22928: K line diffuse in appearance; probably chiefly stellar.

TABLE VII—*Continued*

HD No.	Star	Mag.	Type	Galactic		Int.	Corrected Color Excess, E				R	$\log \frac{N}{N_s}$
				Long.	Lat.	K	Y_1	Y_2	M	Mean		
1	2	3	4	5	6	7	8	9	10	11	12	13
167971†	.....	7.3	Bo	347	0	42.3	....	....	+42	+42	+11	− 2
169454	.....	6.8	Bose	345	− 2	38.8	....	....	+51	+51	+24	+ 5
187811‡	12 Vul	4.9	B5ne	27	− 3	2.8	....	....	− 7	− 7	+ 5	.....
190603	.....	5.7	Bose	37	− 1	25.6	....	....	+34	+34	+22	+ 9
191201	.....	7.1	Bo	41	+ 1	30.8	....	....	+15	+15	− 3	+20
192422	.....	7.1	Bos	44	+ 2	24.2	....	....	+24	+24	+13	+13
193237	P Cyg	4.9	B1eq	43	0	24.1	+32	....	+27	+30	+19	.....
193322§	.....	6.0	O8	45	+ 2	34.0	....	....	+ 9	+ 9	−12	+18
194279‖	.....	7.0	Bos	46	+ 1	50.4	+59	....	+54	+56	+17	+14
194839	.....	7.5	B2se	46	+ 1	44.5	....	....	+52	+52	+19	+18
195592	.....	7.2	B1se	50	+ 2	27.0	....	....	+51	+51	+38	0
198478	55 Cyg	4.9	B2s	53	+ 1	31.7	+26	....	+23	+25	+ 6	+18
200120	59 f¹ Cyg	4.9	B3ne	56	0	5.8	....	....	− 5	− 5	+ 4	+ 9
204172	69 Cyg	5.9	Bo	51	−11	19.4	+ 6	....	0	+ 3	− 2	0
205021	β Cep	3.3	B1	75	+13	5.5	− 8	− 7	− 8	− 8	+ 1	−11
206165	9 Cep	4.9	B2s	70	+ 6	27.6	+16	+20	+19	+19	+ 5	0
208185	.....	8.2	B3	72	+ 7	22.5	+11	+ 6	....	+ 9	0	+10
208392	.....	7.1	B3ne	72	+ 6	30.9	+10	+12	+12	+11	− 7	+14
208947	.....	6.3	B3	74	+ 8	19.0	....	− 5	0	− 2	− 7	+13
210809	.....	7.7	O8	68	− 3	39.2	....	+ 7	....	+ 7	−20	+29
210839	λ Cep	5.2	O6ne	61	+13	22.6	+18	+23	+16	+19	+11	+22
212455	.....	8.4	B3	71	− 2	80.0	....	+20	....	+20	−53	+21
212571	π Aqr	4.6	B1ne	39	−46	8.4	− 2	....	− 1	− 1	+ 5	.....
213420	6 Lac	4.5	B3	65	−14	15.0	....	− 3	0	− 1	− 2	+25
214680	10 Lac	4.9	O9s	65	−18	9.5	− 4	....	− 5	− 4	0	+19
224151	.....	6.1	Bo	83	− 4	21.3	....	....	+16	+16	+ 9	+ 8

† HD 167971: Spectrum almost continuous at $H\beta$. Bright lines other than those of H suspected.
‡ HD 187811: K line diffuse; probably stellar.
§ HD 193322: 6.0–8.2, 2".7. The color measure probably includes the companion.
‖ HD 194279: Brightest star in NGC 6910.

stars with large color excess. As previously remarked, the bright lines themselves have a negligible effect on the measured color, which is wholly influenced by the distribution of energy in the continuous spectrum.

Table VIII gives the relevant particulars of the Be stars observed. The character of emission is from the data of Struve[21] and McLaughlin[22] and from examination of the present plates. In the first group, showing strong emission at $H\beta$, the mean residual color excess, R, is

[21] *Astrophysical Journal*, **73**, 94, 1931.

[22] *Publications of the Observatory of the University of Michigan*, **4**, 175, 1932.

+17.7 (six stars). For the four stars with medium-strong emission, the residual falls to +8.5, and to +5.3 for twelve stars with weak or

TABLE VIII

BRIGHT-LINE STARS

Star	R	Character of Emission		Character of Absorption Lines	Rotation [Struve]
		$H\beta$	$H\gamma$		
Group I. Strong Emission at $H\beta$					
γ Cas	+20	V. broad double	V. broad double	n	10
X Per	+26	Double	Double	nn	.....
χ Oph	+28	Broad	Double	.....	6
HD 58050	− 1	Broad	Broad	ns	2
11 Cam	+14	Narrow single	Narrow single	.....	2
P Cyg	+19	.....	.....	.....	0
Group II. Moderate Emission at $H\beta$					
π Aqr	+ 5	Broad double	Almost continuous	.....	10
ζ Tau	− 2	Broad double	.....	.....	7
o Cas	+ 7	Double	.....	n	.....
HD 169454	+24	Bright red edge	Faint red edge	s	.....
Group III. Weak or No Emission at $H\beta$					
23 Tau	+12	Invisible	.....	.....	10
59 f^1 Cyg	+ 4	Broad double	.....	.....	9
κ Dra	+10	Narrow double	.....	.....	8
12 Vul	+ 5	Invisible	.....	n	.....
HD 208392	− 7	.....	.....	n	.....
9 (α) Cam	− 3	?	.....	.....	4
HD 13854	−19	Red edge?	Red edge?	s	.....
HD 14134	−11	Faint red edge	Faint red edge	s	.....
HD 194839*	+19	Red edge	.....	s	.....
HD 195592	+38	.....	.....	s	.....
χ^2 Ori	− 6	Absorption only	.....	ss	1
HD 190603	+22	Red edge?	.....	ss	.....

* This is a previously unrecorded bright-line star.

no emission at $H\beta$. We may note also that for λ Cephei, O6ne, with bright lines other than those of hydrogen, $R = +11$; and that for ζ Ophiuchi, which from the exceptionally diffuse character of the lines

in its spectrum may be surmised to have passed through an emission phase, $R = +18$.

We have here evidence either for an abnormally low effective temperature for the Be star or, more probably, for a selective scattering of its light in an extensive atmosphere surrounding it and presumably originally cast out from it. On Struve's theory of the origin of the bright lines in matter ejected from stars in rapid rotation, the scattering atmosphere might be expected to form an equatorial ring.[23] When seen through this ring, the star would appear reddened, but if viewed along the axis of rotation, the reddening would be much reduced or non-existent. In the first case the bright H lines should be broad and double; in the second, narrow and single.

The evidence for excess redness of stars with double emission over those with single is inconclusive, since only one typical star of the latter category is included. For this star, however, $R = +14$, and examination of Table VIII, in which the stars in each group have been arranged roughly in the order of the widths of their emission lines, does not offer much support for the suggestion.

In group III the order of the stars has been determined by Struve's values of the axial rotation, on an arbitrary scale of 0:10, which he is able to correlate with emission width. Line sharpness has been used where no rotation is available.

All the Be stars in Table VIII, also λ Cephei and ζ Ophiuchi, have been omitted from the succeeding discussion.

A preliminary search for a color and absolute-magnitude effect among the normal early B stars has been made by separating the diffuse H-line stars from the sharp and counting the numbers of each with positive and negative R values. We find the results as tabulated, indicating that any such effect is small enough to be masked by the scatter.

	Positive R	Negative R
No. of s-line stars	14	11
No. of n-line stars	8	6

LATITUDE AND LONGITUDE EFFECTS

To see whether, with increasing distance from the galactic plane, there is any systematic change in the calcium-intensity and reddening

[23] Elvey, *op. cit.*, p. 308.

relation, the observed galactic latitudes have been reduced to the value they would have if all the stars were distant 100 parsecs. Edwards' spectroscopic[24] and mean absolute magnitudes were used. A plot of reduced latitude against R reveals no effect at all up to latitude 15°, corresponding to 26 parsecs distance from the plane; but for two stars in reduced latitudes 17° and 20°, R is -9 and -10, respectively, whereas the mean for the rest is $+0.7$.

In studying space scattering for distant stars, it is convenient to discard R and use instead the reddening per unit of K intensity, denoted by S. From Figure 6 it appears that the color excess on the Madison scale corresponding to zero distance is -15; consequently, in the units of Table VII, $S=(E+15)/K$. Stars with K <10 have been omitted, as for them the errors become disproportionate. Those with galactic latitudes greater than 20° also were not considered.

Stebbins and Huffer have found large variations in space reddening with longitude l. No such marked oscillations of S with l are apparent in the present somewhat meager data; and when we divide the stars into two sections, one toward the galactic center, the other away from it, no appreciable difference in the relative density of scattering material and absorbing calcium atoms can be detected.

	Mean S	Mean residual
15 stars, Scorpio-Cepheus	10.0	2.8
11 stars, Cassiopeia–Canis Major	11.8	3.8
or, with ζ Persei omitted,	10.5	2.5

STAR DENSITY AND REDDENING

Elvey and Mehlin[25] in their study of the Cepheus region found that the majority of their red stars were in the apparently obscured areas, whereas Stebbins and Huffer[26] state that the stars with the largest color excess are just as likely to be in rich regions as in sparse.

When we remember that the total visual obscuration is about equal to the color excess, we see that for a much-reddened star, say with $M=50$, or approximately 1 mag. color excess on Seares's giant-star scale, the effect of any material *between the earth and the star*, in which Rayleigh scattering alone occurs, will be to decrease by ap-

[24] *Monthly Notices of the Royal Astronomical Society*, **87**, 364, 1927.

[25] *Astrophysical Journal*, **75**, 354, 1932. [26] *Loc. cit.*

proximately 1 mag. the brightness of all stars beyond. This means that in a star count to the apparent magnitude which has a mean distance equal to that of the star in question, the number of stars would be reduced more than 50 per cent by space absorption. Nonselective absorption, heavy obscuration beyond the star, and spatial irregularities of stellar distribution will naturally spoil any close correlation, but a statistical dependence should remain.

As a test for any such connection between reddening and field richness, star counts were made in circular areas surrounding the stars observed. The counts, to magnitudes 15.1, 13.8, 10.3, and 8.5, were made on Franklin Adams and *B.D.* charts and on Stuker's star atlas, and embraced areas of, respectively, 0.1, 0.1, 2.0, and 38.5 square degrees. For each set the relation between log N_c, where N_c indicates the number of stars counted per square degree, and galactic latitude was compared with the Mount Wilson results from the Selected Areas.[27] The residuals log (N_c/N_s) were found for each star and so adjusted that the algebraic means of the values derived from the different charts became coincident. Finally, the residuals corresponding to the four limiting magnitudes were examined for each star. If the variation was small, the figure entered in column 13 of Table VII is the mean value of log (N_c/N_s); if the residuals for the fainter magnitudes decreased, the defect was attributed to space absorption beyond the nearer stars and log (N_c/N_s) for the brighter magnitude limits, corresponding more closely to the distance of the star under consideration, is given. When the residual increases with the magnitude limit, real fluctuations in star density and magnitude errors on the charts are involved, and a mean has again been taken.

The results, in Table IX, are admittedly rough, but give a clear indication of an effect in the expected direction. That is to say, as log (N_c/N_s) decreases, indicating an increasing amount of obscuring material in the direction of the star, the space reddening, S, becomes greater. The decrease in scatter with increasing field richness is worthy of notice.

The stars HD 212455, 210809 are 3° apart in the richest part of the northern Milky Way in Cepheus. The first, with the strongest

[27] Seares, Van Rhijn, Joyner, and Richmond, *Mt. Wilson Contr.*, No. 301; *Astrophysical Journal*, **62**, 362, 1925.

observed K line, is probably at twice the distance of the second. Since S is, respectively, 0.44 and 0.56, the weak reddening is probably sensibly uniform in this direction. For HD 191201, in the richest part of the Cygnus cloud, S is considerably larger (0.98). For four stars within $3\frac{1}{2}^\circ$ of each other in another fairly rich Cygnus field, the data are as in Table X.

The second star, of intermediate distance, apparently in the middle of a small bright nebula, is evidently viewed through an excep-

TABLE IX

$\log \frac{N_c}{N_s}$	<0	0 to +0.09	+0.10 to +0.20	>+0.20
S	1.30	1.11	1.06	0.84
Mean residual	0.40	0.27	0.31	0.17
Number of stars	6	7	7	5

TABLE X

Star	Mag.	K Int.	$\log \frac{N_c}{N_s}$	S
HD 192422	7.1	24	+0.13	1.61
193322	5.8	34	.18	0.70
194279	7.0	50	.14	1.41
194839	7.5	45	+0.18	1.50

tionally transparent lane. In the region around ζ Persei, on the other hand, there appears to be a concentrated cloud of reddening material. For this star, $S = 2.5$, and for X Persei, only 1° distant, we find a residual color excess of +26, which is abnormally large, even for a bright-line star, while for ξ Persei, 5° from ζ, S has dropped to 1.6. Further, in the immediate vicinity of o Persei, within 4° of ζ Persei, Seares and Hubble have found numerous faint stars which have large values of the color excess (unpublished results).

We may conclude, therefore, that, although in apparently transparent areas reddening is generally both less in amount and subject to smaller fluctuations than in obscured regions, there is little uniformity when one region is compared with another, and that in most

parts of the Milky Way there is great irregularity in the distribution of scattering particles.

The most profitable line for further work on the distance and color-excess relation would seem to be the study to faint magnitudes of small homogeneous fields, some in dust-free regions in order to find the degree of constancy of selective scattering over large distances, and some in front of known areas of distant obscuration where the rise of S with K intensity as the dark nebulosity is approached would reveal more clearly the connection between the latter and the reddening material.

It is a pleasure to express my indebtedness to Dr. Adams and to many others at Mount Wilson for their interest and helpful advice on numerous points connected with the work. Dr. Dunham in particular rendered invaluable assistance in the design of the standardizing attachment. I am particularly grateful to Dr. Stebbins and Dr. Huffer for providing me with their color excesses in advance of publication. Finally, I wish to thank the night assistants for their part in obtaining good spectra.

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MOUNT WILSON OBSERVATORY
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THE B BAND OF OXYGEN IN THE SPECTRUM OF MARS[1]

By WALTER S. ADAMS and THEODORE DUNHAM, Jr.

ABSTRACT

Ten spectrograms of Mars covering the region of the B band of oxygen were obtained in the first order of the 9-foot grating spectrograph at the coudé focus of the 100-inch reflector during the autumn and winter of 1932–1933. During the interval covered by the observations, the radial velocity of the planet relative to the earth varied from −13.8 to +12.6 km/sec. The linear scale of the spectrograms was 5.6 A/mm.

Measures of the wave-lengths of about thirty oxygen lines on each of the ten spectrograms relative to lines of solar origin showed a mean displacement of but 0.0002 A from the position of the telluric oxygen lines. The contours of the oxygen lines were also investigated with the microphotometer and compared with the theoretical contours on the assumption of a ratio of 1:1000 in the relative abundance of molecules of free oxygen in the atmospheres of Mars and the earth. The final conclusion is that the amount of oxygen in the atmosphere of Mars is probably less than one-tenth of 1 per cent of that in the earth's atmosphere over equal areas of surface.

The detection of band lines arising from the presence of oxygen in the atmosphere of Mars is difficult on account of the strong overlying lines of telluric oxygen. Under suitable conditions, this difficulty may be overcome in two different ways.

First, the intensity of the oxygen band in the spectrum of Mars may be compared photometrically with the intensity of the band in a spectrum of the moon taken at the same altitude above the horizon. This was done in 1909 by F. W. Very, who used single-prism spectra of the B band taken by V. M. Slipher. Very concluded that the band was 24 per cent stronger in the spectrum of Mars than in that of the moon and attributed the difference to Martian oxygen.

The second method depends on the Doppler shift of Martian oxygen lines relative to the corresponding telluric lines when the planet is near quadrature. Percival Lowell suggested in 1905 that under these conditions each telluric oxygen line should be blended with a Martian oxygen line in its wing, with the result that its measured wave-length should differ slightly from the normal value. Slipher attempted to apply the method to the B band, but was unable to obtain a definite result. In 1910 W. W. Campbell and S. Albrecht,

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington,* No. 488.

using higher dispersion on the a band, concluded that if oxygen were present in the atmosphere of Mars its amount must be small.

The a band in the spectrum of Mars was also photographed by Adams and St. John in 1926, with a dispersion of six prisms. The linear scale of the single spectrogram obtained was about 11 A to the millimeter and measures with a microphotometer pointed to a slight displacement of the oxygen lines. With allowance for the relative paths in the atmospheres of the two planets, the measurements indicated the presence of about 16 per cent as much oxygen above unit area on Mars as on the earth.

Recently Dr. Mees at the Research Laboratory of the Eastman Kodak Company has produced a photographic emulsion having a narrow band of sensitivity with a maximum near λ 6800. This emulsion has made it possible for us to photograph with high dispersion the strong B band of oxygen in the spectrum of Mars, which is much more favorable for an investigation of this character than the faint a band of shorter wave-length. The instrument used was a plane-grating spectrograph of 9-foot focal length, placed in a fixed position at the coudé focus of the 100-inch telescope. The linear scale of the spectrograms was 5.6 A per millimeter in the first order, and the definition was good. The average exposure time on the planet was three hours.

Ten spectrograms of the B band in Mars were obtained with this instrument: one in November, 1932; another in January, 1933; two in March; four in April; and two early in May. During this interval the radial velocity of the planet relative to the earth changed from -13.8 to $+12.6$ km/sec. In this region of the spectrum a velocity of 13.8 km/sec. would correspond to 0.32 A, or to a linear displacement on the spectrograms of 0.057 mm. Since this shift is about one-third of the average distance between the components of the conspicuous pairs in the tail of the band, the cores of lines due to oxygen in the atmosphere of Mars should be completely resolved. No trace of any such lines could be seen, however, and measurements of the telluric lines were undertaken to learn whether slight shifts due to the presence of possible very faint unresolved Martian components could be detected.

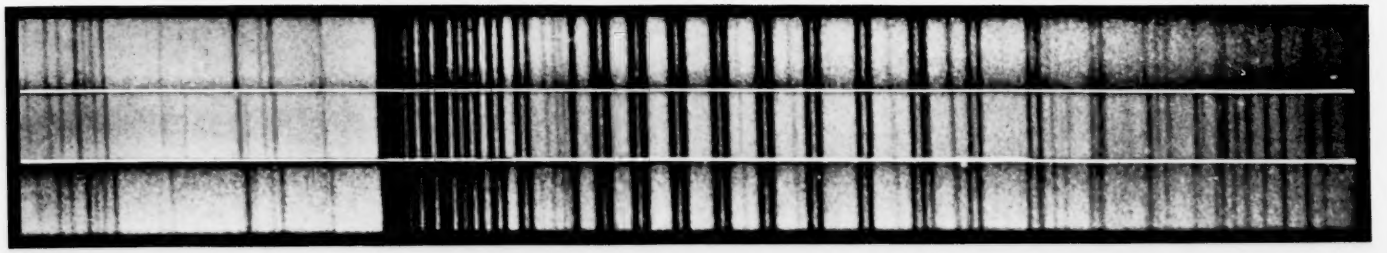
Since the arc spectrum of iron which is used for comparison pur-

poses on spectrograms taken with the coudé instrument shows only a few faint lines in the region of the B band, solar lines were adopted as standards for the measurement of the oxygen lines. This method has the advantage of comparing absorption lines with absorption lines photographed simultaneously, and of utilizing the wave-lengths of the *Revised Rowland Table* throughout. Owing to the radial velocity of Mars, the lines of oxygen, if wholly terrestrial in origin, would be displaced by amounts corresponding exactly to the velocity of Mars,[2] but with reversed sign; but by smaller amounts, if any Martian oxygen is involved. Since the scale of the spectrograms is not strictly constant, owing principally to the color-curve of the lens in the spectrograph, the reduction of the measures requires the use of some form of interpolation formula. An equation of the Hartmann type involving three standard lines has been found satisfactory, the computed wave-lengths of the oxygen lines being corrected by means of a curve derived from other lines of known wave-length. Direct interpolation with a scale factor derived from short intervals along the spectrograms has also been used to some extent, and in general the wave-lengths thus obtained agree closely with those computed from the Hartmann formula. At best, the distribution of solar lines in this region of the spectrum is not good for comparison purposes, but we believe that the methods of reduction employed have reduced the errors from this source to small quantities. On the average, thirty oxygen lines in the B band have been measured on each spectrogram.

The final results, based in most cases upon measurements by two observers, are shown in the accompanying table. The calculated radial velocity of Mars, converted into fractions of an angstrom unit for direct comparison with the measured displacements of the oxygen lines, is given in the third column. The difference between the measured and the calculated values is in the last column. The method of reduction, which is based upon solar lines in the spectrum of

[2] This velocity is made up of the three components, Mars-earth, Mars-sun, and rotation of earth. Solar lines in the spectrum of Mars would be displaced by an amount corresponding to the algebraic sum of all three components, but oxygen lines in the spectrum of Mars by an amount corresponding to the sum of the first and third terms. The maximum velocity of Mars relative to the sun for any of the ten spectrograms here considered is +1.5 km/sec.

PLATE VII



SPECTRUM OF THE B BAND IN MARS AND THE SUN

a) Mars ($V = -13.75$ km/sec.)
b) Sun
c) Mars ($V = +12.42$ km/sec.)

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Mars, is such that the presence of oxygen lines in Mars would in all cases give smaller numerical displacements for the oxygen lines measured on the spectrograms. Hence a persistent negative sign in these differences would point to the existence of Martian oxygen.

SUMMARY OF OXYGEN LINES IN MARS

Plate	Calc. V		Meas. V A	Diff. A
	Km/Sec.	A		
Coudé 579	−13.75	0.316	0.295	−0.021
594	−13.21	.304	.298	− .006
613	+ 3.55	.082	.077	− .005
618	+ 3.73	.086	.085	− .001
622	+10.30	.237	.232	− .005
627	+10.48	.241	.256	+ .015
632	+10.81	.248	.242	− .006
637	+10.86	.250	.252	+ .002
649	+12.42	.286	.298	+ .012
653	+12.60	0.290	0.303	+0.013
Mean difference				−0.0002 ±0.0024

If the lines measured on the spectrograms may be considered as blended lines consisting of two components, one due to telluric oxygen and the other to Martian oxygen displaced by an amount corresponding to the relative motion of Mars and the earth, the order of the intensity of the Martian component may be calculated from the measures. The mean difference shown by the ten spectrograms is −0.0002 A, which would indicate a Martian component considerably less than one-tenth of one per cent as strong as the telluric component. This quantity is far below the accidental error of measurement, the probable error of the mean being ±0.0024 A. A value of 0.0024 A, however, would indicate a Martian component only 0.8 of one per cent as strong as the telluric component, and the number of oxygen molecules producing the Martian line would be considerably less than 0.01 of one per cent as great as the number producing the telluric line.

Further, if oxygen is present in appreciable amount in the atmosphere of Mars, it should produce an asymmetry in the lines of the telluric B band due to Martian components, in the violet wings

when the planet was approaching in November, 1932, and in the red wings when the planet was receding in May, 1933. A comparison of the contours of these lines on the plates taken at these times should bring out such an asymmetry. Plate VIII shows tracings obtained with the registering microphotometer of two spectrograms corresponding to the extreme range in radial velocity.

Since the components of each pair of lines in the tail of the B band are separated by only about 0.9 A, the wings overlap, and it is im-

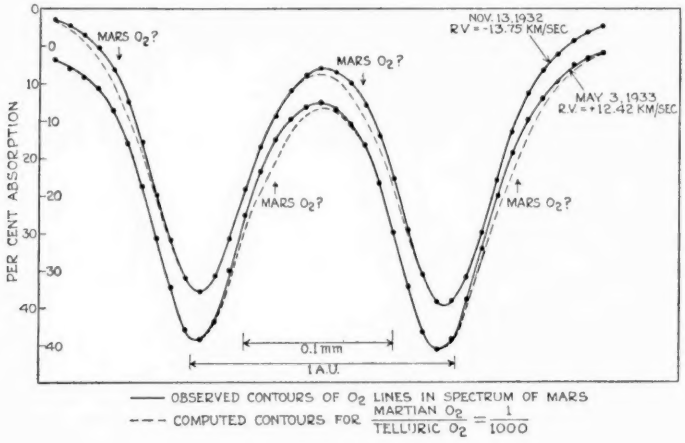
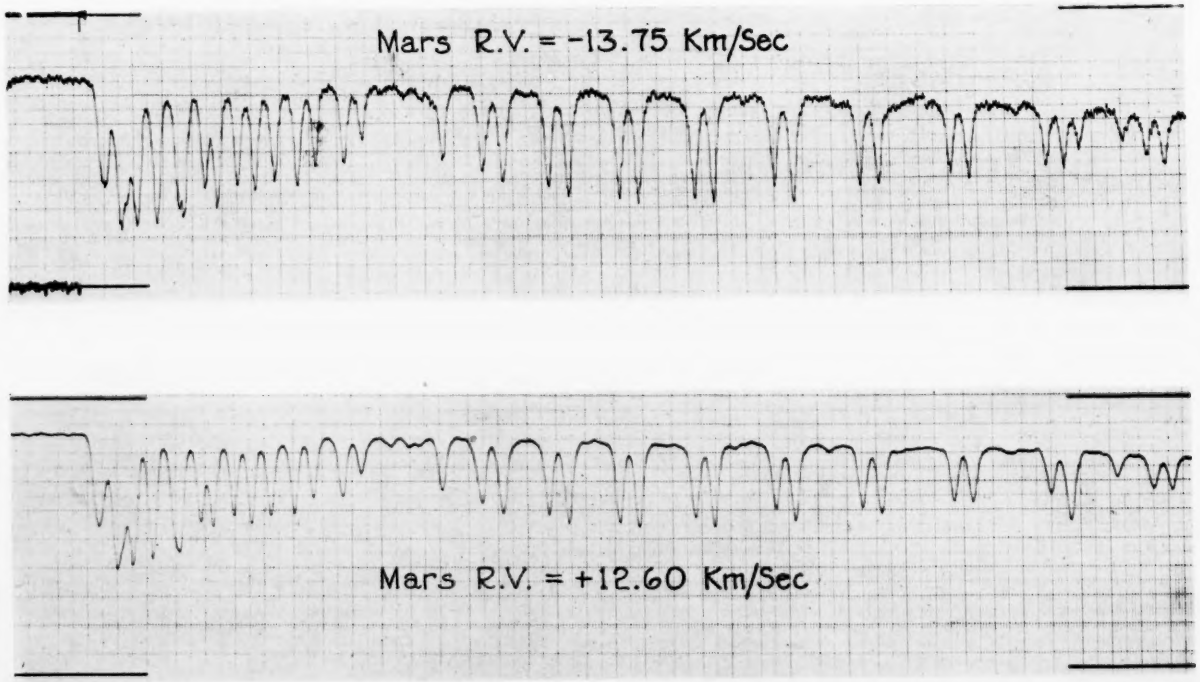


FIG. 1

possible to study the lines individually. They must therefore be studied in pairs. Accidental errors have been reduced by averaging the contours of six pairs of lines on each plate. Figure 1 shows the contours on November 13, 1932 ($V = -13.75$ km/sec.) and on May 3, 1933 ($V = +12.42$ km/sec.), the curves being displaced vertically for comparison. Before averaging the contours, the abscissae of individual pairs of lines were adjusted to take account of the slightly different separations of the components of different pairs. As the two plates were not taken at exactly the same zenith distance, the ordinates have been adjusted to make the total absorption equal in the two cases.

Since any asymmetry produced in these curves by Martian oxygen would affect the violet wings of one curve and the red wings of the other curve, a direct comparison of the curves should show a

PLATE VIII



MICROPHOTOMETER TRACINGS OF THE B BAND IN THE SPECTRUM OF MARS

doubled effect. No such asymmetry is apparent from an inspection of Figure 1. In order to bring out differences that do exist in the shapes of the two curves, the differences in percentage absorption are plotted on an enlarged scale in Figure 2 for each 0.01 mm on the plate (0.056 A). The differences were taken after sliding one curve on the other longitudinally until the best fit possible was obtained.

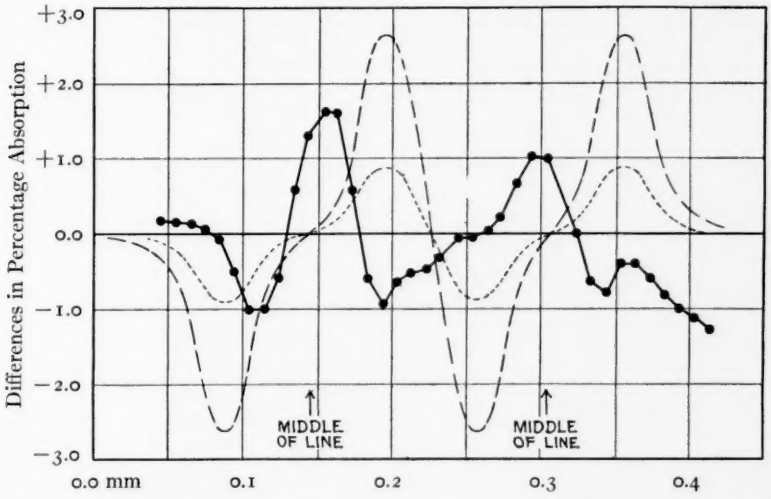


FIG. 2.—Differences in percentage absorption within pairs of oxygen lines shown in Figure 1, expressed as absorption: Mars receding (May 3, 1933, $V = +12.42$ km/sec.) *minus* Mars approaching (November 13, 1932, $V = -13.75$ km/sec.)

Full line: Observed differences in percentage absorption.

Dashed line: Theoretical differences for $\frac{\text{Martian } O_2}{\text{Telluric } O_2} = \frac{1}{1{,}000}$

Dotted line: Theoretical differences for $\frac{\text{Martian } O_2}{\text{Telluric } O_2} = \frac{1}{10{,}000}$

The observed differences between the two contours do not at any point exceed 1.5 per cent of the background intensity. There is no certain systematic agreement between the observed and theoretical curves, and hence no evidence for the existence of Martian oxygen.

An attempt has been made to estimate the contours these lines would have if the ratio of oxygen in the atmospheres of the two planets were 1:1000. Since the Martian path is approximately twice that of the telluric path, and since the total absorptions of spectral lines may be taken to vary as the square root of the number of molecules

concerned, we should expect an integrated absorption equal to 4.5 per cent of that of the lines in the telluric band. The half-width of the monochromatic pattern of the spectrograph used is known to be about 0.33 A, and the displacement of the centers of the hypothetical Martian lines on the two dates in question is known from the computed velocity of the planet. The approximate contour for the blended oxygen lines can therefore be calculated. The computed contour for an abundance ratio of 1:1000 is shown in Figure 1 as a dotted line.

The differences in the ordinates of the two theoretical curves for Martian O_2: Telluric $O_2 = 1:1000$ are shown in Figure 2 by the dashed curve of larger amplitude. The dotted curve of smaller amplitude represents a ratio of 1:10,000 for the amounts of oxygen on the two planets. There is no indication of any significant similarity between the observed and the theoretical curves. The excursions of the observed curve nowhere represent differences in absorption between the two contours of more than 1.5 per cent, and are probably due to slight differences in focus of the spectrograph on the two dates in question. Judged by the magnitude of the observed fluctuations, it is highly probable that Martian oxygen amounting to 0.001 of the telluric oxygen could be detected with the present resolution, but that the detection of one-tenth of this amount would be uncertain.

It is of interest to estimate the displacement in wave-length to be expected from measures made by setting a wire on the apparent central wave-lengths of the oxygen lines for a ratio of Martian to telluric oxygen of 1:1000. In making settings on an asymmetrical line some observers are influenced more by the core and others more by the wings. For the present purpose we shall assume that the setting bisects the integrated absorption of the line. The differences in the abscissae of the two theoretical curves in Figure 1 indicate that for an oxygen ratio of 1:1000 and a radial velocity of 13.09 km/sec. (the mean of the two observations) we should expect a measured displacement of the line as a whole amounting to 0.010 A. This value is considerably greater than the probable error of the present measures, and the measures themselves give no indication of any such displacement.

The observed differences between the contours for November, 1932, and May, 1933, which are plotted in Figure 2 were obtained after sliding the curves horizontally to obtain the best fit. Under the assumed conditions, a displacement of the November curve in the direction of increasing wave-length amounting to twice the shift for each curve, or 0.020 A in all, might therefore have been expected. In order to include with certainty the correct match in wave-length, four observed difference-curves similar to Figure 2 were plotted, corresponding to the following relative positions of the November and May curves:

1. Best fit of the two curves
2. November curve displaced 0.020 A toward shorter wave-lengths
3. November curve displaced 0.040 A toward shorter wave-lengths
4. November curve displaced 0.020 A toward longer wave-lengths

In order to avoid confusion, only the first of these curves is reproduced in Figure 2. The others show, as is to be expected, a progressive change in shape, but none indicates any systematic agreement with the theoretical curves shown by dashed lines. We appear, therefore, to be justified in concluding that the observed shapes of the oxygen lines give no indication whatever of Martian oxygen, whereas an oxygen ratio of 1:1000 would cause a readily detectable asymmetry.

The A band of oxygen with its head near λ 7600 includes the strongest oxygen lines in the whole solar spectrum that can be studied photographically, and hence should be particularly favorable for investigation because of the increased intensity of possible Martian components. This band has the further advantage that in fully exposed solar spectra it shows the isotope lines of $O^{16}-O^{18}$, which are due to a definite number of absorbing molecules about 0.06 per cent as numerous as those producing the principal lines and which thus afford a basis for quantitative comparison of the intensity of Martian oxygen lines. The difficulties in obtaining high-dispersion spectrograms of sufficient density in this region of the spectrum on so faint an object as Mars are considerable. As yet we have but a single spectrogram, which was taken with the aid of a cylindrical lens placed close to the photographic plate to reduce the exposure time. This photograph agrees with those of the B band in

showing no certain evidence of the presence of Martian components; but plates of greater density and higher contrast are required for a definite conclusion.

A few lines due to water vapor, lying to the red of the oxygen lines in the tail of the B band, appear on our spectrograms. The sensitivity of the plate falls off rapidly in this region and the focus of the spectrograph changes rapidly. The few measures made indicate displacements slightly smaller than those calculated from the velocity of the planet, as might be expected were Martian components present, but these measures are entitled to little weight. We hope to undertake further investigations of the water-vapor lines near λ 7200 and λ 8200, where much stronger lines are available.

The chief conclusion to be drawn from this study with high dispersion of the B band of oxygen in the spectrum of Mars is that the amount of free oxygen present in the atmosphere of the planet must be extremely small, certainly less than 1 per cent, and probably less than 0.1 per cent, of that present in the earth's atmosphere over equal areas of the surface.

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MOUNT WILSON OBSERVATORY
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MOLECULES IN THE SUN AND STARS[1]

By HENRY NORRIS RUSSELL[2]

ABSTRACT

Molecular constants, derived mainly from band spectra, have been collected for about thirty diatomic molecules likely to occur in the sun and stars, and the *dissociation constants* calculated. From these and the writer's determinations of the abundance of elements in the sun and the pressures prevailing above the disk and the spots, the *amounts of various compounds* have been calculated. The results are in good agreement with observations. The prominence of *hydrides* results from the great abundance of hydrogen and affords an *independent proof* of this abundance. The moderate weakening of the bands of non-metallic compounds in passing from the spot to the disk arises from dissociation; the great weakening for metallic compounds, from the combined effect of dissociation of compounds and ionization of metallic atoms. The *abundance of F*, estimated from *SiF*, is comparable with that of *N* or *Na;* that of *B* is much smaller. Some of the most abundant molecules—H_2, N_2, *CO*—have strong bands only beyond λ 3000. The bands of H_2 in the visible region come from states of excitation potential exceeding 11.7 volts and must be too faint to observe. They would be weakened in the spots.

For *stars similar in composition to the sun* (with oxygen in excess of carbon), the amounts of *H*, *C*, *N*, *O*, *Ti*, and their compounds have been calculated, with the aid of the writer's recent determinations of atmospheric extent and pressure in giants and dwarfs, and are illustrated by diagrams. The observed behavior of the bands is closely represented. The maxima of *CH* and *CN* near class K arise from a shift of equilibrium at low temperatures toward the more tightly bound compound *CO*, and are quantitatively explained, as is the sensitiveness of the *CN* bands to absolute magnitude.

The predicted maximum intensity of *TiO* is considerably higher in giants than in dwarfs (agreeing with observations). The S stars belong to the oxygen group and probably contain more *Zr* than normal.

A possible mechanism is suggested for the excitation of metallic emission lines in long-period variables by the dissociation of compounds, and it is shown that the weakening of the sodium lines at minimum cannot be explained by the conversion of the metal to oxide or fluoride.

The predicted properties of stars with an excess of carbon over oxygen closely resemble those of classes R and N (as suggested by R. H. Curtiss). The disappearance of the violet cyanogen bands in the later N's is attributed to the absence of continuous background, since the red *CN* system and the band near λ 4600 do not weaken.

Dwarf stars with excess of carbon should show spectra of classes R or N. Carbon stars are so rare among giants that it is not remarkable that no carbon dwarf has yet been found.

Stars of intermediate composition should exhibit spectra likely to be classed as peculiar examples of class M, R, or N.

1. It has long been known that faint lines in the solar spectrum, stronger lines in sun-spots, and huge bands in red stars are produced by molecules. The first attempt at quantitative discussion of the

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 490.

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dissociation of such compounds appears to be that of Atkinson[3] for the titanium bands. Later, G. Piccardi[4] discussed H_2, and A. Christy[5] *TiO;* but the first comprehensive treatment of the problem is due to Wildt,[6] who laid down the main lines of all future work, reached many important conclusions, and would probably have gone still farther had good-enough numerical data then been available. Special cases have since been treated by Swings,[7] Swings and Struve,[8] and Wurm.[9]

The present work (upon which a preliminary note[10] appeared in 1931) utilizes recent and more accurate data on the composition and pressure of solar and stellar atmospheres to calculate the abundance of various compounds, especially those which show conspicuous band spectra in the accessible region. The results are in close agreement with observation, and show that the problem of band intensities in stellar spectra, like that of line intensities, is solved to the first approximation.

2. The dissociation of any diatomic molecule is governed by an equation of the form

$$p(A)p(B)=K(AB)p(AB)\,, \qquad (1)$$

where $p(A)$ and $p(B)$ are the partial pressures of the free atoms, $p(AB)$ that of the compound, and $K(AB)$ a rapidly varying function of the temperature. This equation is identical in form with the equation of ionization. If n elements are simultaneously undergoing (first) ionization, we have to deal with $2n+1$ quantities—the numbers of neutral atoms, ions, and electrons—connected by $n+1$ equations expressing constancy of composition and n others of type (1) defining the ionization. But if the n kinds of neutral atoms may combine *inter se*, we must discuss the abundance of n elements and

[3] R. d'E. Atkinson, *Monthly Notices of the Royal Astronomical Society*, **82**, 396, 1922.

[4] *Memorie della Società Astronomica Italiana*, **4**, 379, 1929.

[5] *Astrophysical Journal*, **70**, 1, 1929.

[6] R. Wildt, *Zeitschrift für Physik*, **54**, 856, 1929.

[7] P. Swings, *Monthly Notices of the Royal Astronomical Society*, **92**, 140, 1931.

[8] P. Swings and O. Struve, *Physical Review*, **39**, 142, 1932.

[9] K. Wurm, *Zeitschrift für Astrophysik*, **5**, 260, 1932.

[10] *Publications of the Astronomical Society of the Pacific*, **43**, 396, 1931; *Carnegie Institution of Washington, Year Book*, No. 30, 192, 1931.

$\frac{1}{2}n(n-1)$ possible compounds, and have $\frac{1}{2}n(n+1)$ equations. Both problems are determinate, but that of dissociation is the more complex.

When we pass to mean values for an atmosphere of considerable depth, the complications become very great. It is practically necessary to start by assuming a constant temperature. In the case of ionization it is known that this assumption gives a good approximation, provided the effective temperature of the photosphere is used.[11] Whether the same is true for dissociation is not yet known, but it may at least be hoped that the errors are not serious.

It is also permissible to take the pressure as uniform, and equal to half the value at the "bottom" of the atmosphere.[12] This procedure gives remarkably good values of the mean ionization over a wide range of conditions. Here the two problems are exactly parallel. The equations for an isothermal atmosphere consisting (at low temperatures) of inert gas and two dissociable diatomic compounds with a common ion are precisely similar to those for an atmosphere of inert gas and two ionizable elements, and the writer's discussion[13] may be taken over bodily. Dissociation in the presence of excess of one constituent resembles ionization in the presence of completely ionized matter, with the pressure of the more abundant constituent taking the place of the electron pressure. Milne's approximation is good, except that it gives too low a value for the amount of dissociation when this is just beginning (which is of no practical importance). When dissociation is nearly complete, his rule holds good for any number of compounds.

If p is the total pressure, we have then, approximately, $p_1 = E_1 p$, $p_2 = E_2 p$ where E_1 and E_2 are abundance factors. The abundance of the compound at the p-level is $E_{12} = p_{12}/p$, and the total amount above this is proportional to $\int_0^p E_{12} dp = E_1 E_2 p^2/2K$, so then the mean abundance is $\bar{E}_{12} = \frac{1}{2}E_{12}$.

Ionization of the constituent atoms decreases the factor. If our

[11] A. Pannekoek, *Bulletin of the Astronomical Institutes of the Netherlands*, **3**, 257, 1926.

[12] E. A. Milne, *Monthly Notices of the Royal Astronomical Society*, **89**, 17, 1928.

[13] H. N. Russell, *Mt. Wilson Contr.*, No. 477; *Astrophysical Journal*, **78**, 246 ff., 1933.

component is highly ionized, the partial pressure of its ions will be $E_1 p$. Taking the electron pressure as Ep, we have for the neutral atoms $p_1 = E_1 E p^2 / K_1'$ (where K_1' is the constant for ionization). Then $E_{12} = E_1 E_2 E\, p^3 / K K_1'$; $\overline{E}_{12} = \frac{1}{3} E_{12}$. For partial ionization the factor lies between $\frac{1}{2}$ and $\frac{1}{3}$.

Still following Milne, we may take the thickness of this fictitious atmosphere to correspond to a given optical depth in the real one. The latter is determined mainly by the ionization. The writer's calculations[14] indicate that the amount of material "above the photosphere" is nearly constant for temperatures below 6500°, owing to the interplay of contrary influences, and depends much more on surface gravity than on temperature so long as the latter is below 6500°. We will therefore adopt a fictitious homogeneous atmosphere, of mass per unit area derived from these calculations, at the effective temperature of the star and at a pressure equal to half the weight of this layer per unit area. Though egregiously simplified, this model has proved very useful in the study of ionization, and there are good physical reasons why it is better than it looks.

3. The dissociation constant K, according to Gibson and Heitler,[15] is defined for molecules for two similar atoms by the equation

$$\ln K = \frac{D}{kT} + \tfrac{3}{2} \ln \frac{(\pi m k T)}{h^2} - \ln \frac{(8\pi^2 I)}{h^2} + \ln \left(1 - e^{-\frac{h\nu}{kT}}\right) + \ln 2 + 2 \ln g , \qquad (2)$$

where D is the dissociation potential, I the moment of inertia of the molecule, ν the limiting vibrational frequency, m the atomic mass, and g the weight of the ground state of the atom. This formula is fully confirmed by the observed data for iodine. When the atoms are unlike, m must be replaced by $M = m_A\, m_B / m_{AB}$, and g^2 by $G = g_A\, g_B / g_{AB}$. If r is the equilibrium distance of the nuclei and ω the "frequency" in cm^{-1}, $I = Mr^2$, $h\nu/k = 1.43\omega$.

It is found empirically that $(1 - e^{-x})/xe^{-0.46x}$ lies between 0.99 and 1.01 if $0 < x < 1.2$, increasing to 1.08 when $x = 2$. The lower range includes almost all cases of astrophysical interest.

[14] *Ibid.*, pp. 284, 285.

[15] *Zeitschrift für Physik*, **47**, 470, 1928.

Making these substitutions, measuring p and K in atmospheres, M in chemical units (1.65×10^{-24}g), r in angstroms, and D in volts, (2) becomes

$$\log_{10} K = -\frac{5040D}{T}+\tfrac{1}{2}\log T-\frac{0.286\omega}{T}+\tfrac{1}{2}\log M-2\log r +\log\omega+\log G+0.24\,. \tag{3}$$

This equation represents the values of log K calculated by Johnston and Walton[16] for O_2 (with detailed consideration of many refinements) from 1400° to 5000°, with an average error of 0.012 and a maximum of 0.026—which are negligible for astrophysical purposes.

4. The constants for molecules which may occur in stellar atmospheres are given in Table I. The data are from Jevons' *Report on Band-Spectra of Diatomic Molecules* (Cambridge, 1932), pages 268–291, with additions as noted.

The dissociation potentials are difficult to determine and are accurately known only for H_2 and O_2. Others are still debated, for example, Kaplan[17] derives 9.0 ± 0.1 for N, and Lozier[18] 7.90 ± 0.02, Kistiakowsky[19] adopts 8.0 for CN, and Mulliken[20] 7.1—the differences arising mainly from differences of theoretical interpretation. The higher or lower pair of values should be used together[21] to avoid inconsistency with thermochemical data; the latter will be used here, without suggesting a critical verdict on the question.

Uncertain values of this sort are included in square brackets in Table I. Those in parentheses are less reliable. Many of them have been found by extrapolation of the lower vibrational levels by the method of Birge and Sponer,[22] which often gives estimates too high by 20 per cent or more.

In computing G the sum of the weights of all components of the

[16] *Journal of the American Chemical Society*, **55**, 187, 1933.

[17] J. Kaplan, *Physical Review*, **42**, 97, 1932.

[18] W. W. Lozier, *ibid.*, **44**, 575, 1933.

[19] *Journal of Physical Chemistry*, **1**, 432, 1933.

[20] *Reviews of Modern Physics*, **4**, 73, 1932.

[21] I owe this suggestion to Professor Linus Pauling.

[22] See W. Jevons, *Report on Band-Spectra of Diatomic Molecules*, p. 193, 1932.

TABLE I

MOLECULAR CONSTANTS

Mol.	ω	r	D	G	M	A	B	log K_{5740}	log K_{4720}
H_2	4371	0.75	4.43*	4	0.50	4.59	23,580	2.36	1.43
BH	(2230)	1.23	[3.45]	12	0.92	4.47	18,000	3.2	2.5
CH	2851	1.13	(4.0)	3	0.92	4.05	21,000	2.3	1.5
NH	(3000)†	1.08	[4.4]‡	8/3	0.93	4.06	23,000	1.9	1.0
OH	3568	0.97	[5.0]‡	3	0.94	4.29	26,200	1.6	0.6
MgH	1493	1.73	[1.41]	1	0.96	2.93	7,500	3.5	3.2
AlH	1681	1.64	3.06	12	0.96	4.11	15,900	3.2	2.6
SiH	(2012)	1.53	(3.0)§	3	0.97	3.65	16,000	(2.7)	(2.1)
CaH	1317	2.02	(1.9)	1	0.98	2.75	10,000	2.9	2.5
CuH	1940	1.46	3.0	4	0.98	3.80	15,700	2.94	2.31
ZnH	1552	1.61	0.91	1	0.99	3.02	5,000	4.03	3.80
O_2	1556	1.20	5.09	27	8.00	5.16	26,200	2.47	1.44
BO	1885	1.20	[6.6]	27	6.50	5.20	33,800	1.2	−0.1
CO	2167	1.15	(10.0)	81	6.86	5.79	51,000	−1.2	−3.2
NO	1906	1.15	[6.1]	6	7.46	4.62	31,300	1.0	−0.2
AlO	977	1.61	(4.1)‖	27	10.1	4.74	21,000	(2.9)	(2.1)
SiO	1240	1.50	(7.9)‖	81	10.2	5.39	40,000	(0.3)	(−1.2)
TiO	1008	1.62	(6.74)¶	21	12.0	4.69	34,000	(0.6)	(−0.7)
ZrO	937	(1.8)**	(7.5)‖	21	13.6	4.59	38,000	(−0.1)	(−1.6)
C_2	1641	1.31	[5.5]	9	6.00	4.57	28,200	1.5	0.4
CN	2069	1.17	[7.1]	18	6.46	5.08	36,400	0.6	−0.8
N_2	2360	1.09	7.90	16	7.00	5.16	40,500	0.0	−1.6
SiF	865	2.26	(5.0)‖,††	9‡‡	11.4	3.95	25,000	(1.5)	(0.5)
SiN	1152	1.53	(6.2)‖	18	9.36	4.65	35,000	(1.1)	(−0.1)
Na_2	159	3.07	[0.76]	4	11.5	2.60	3,900	3.8	3.6
HF	4037	0.86	6.4§§	12	0.95	5.04	33,500	1.1	−0.2

* O. W. Richardson and P. M. Davidson, *Proc. Royal Soc.*, A, **142**, 76, 1933.

† Estimated by plotting ω against r for other hydrides.

‡ Bates, *Physikalische Berichte*, **12**, 2465, 1931.

§ Estimated by L. Pauling (personal communication).

‖ Estimated by Birge and Sponer's approximate formula.

¶ A. Christy, *Astrophysical Journal*, **70**, 1, 1929.

** Estimated from *TiO*; r_0 usually increases by about 0.15 between similar compounds of the first and second long periods.

†† Johnson and Jackson, *Proc. Royal Soc.*, A, **116**, 327, 1927.

‡‡ Character of ground state assumed to be $^2\Pi$ like *Sn Cl*.

§§ From thermochemical data.

ground state has been used. In the last four columns are the coefficients of equation (3), written in the form

$$\log K = A + \tfrac{1}{2} \log T - \frac{B}{T}, \qquad (4)$$

and the values of log K for $T = 5740°$ and $4720°$. Very few of these values are reliable to 0.1, and many are uncertain by 0.5 or more. The differences of the two columns are more accurate. For certain additional reactions, values very kindly computed for the purpose by Mr. F. B. Stitt have been communicated by Professor Linus Pauling, as follows:

$$H_2O = H + OH, \quad \Delta F° = 117{,}700 - 29.7T = -RT \ln K,$$
$$C_2N_2 = 2CN, \quad \Delta F° = 77{,}000 - 36.7T,$$
$$CO_2 = CO + O, \quad \Delta F° = 145{,}700 + T \ln T - 52.74\,T.$$

In the notation of the present paper, these become

$$H_2O, \quad \log_{10} K = 6.50 - 25{,}700/T,$$
$$C_2N_2, \quad \log_{10} K = 8.04 - 16{,}800/T,$$
$$CO_2, \quad \log_{10} K = 11.54 - 0.505 \log_{10} T - 31{,}800/T.$$

5. We may now apply these principles to the sun's atmosphere. The numbers of atoms per unit area, S, on an arbitrary scale have been derived by the writer[23] by calibration of Rowland's intensities. For the idealized homogeneous atmosphere, these are proportional to the partial pressures. For the free electrons log $S = 8.0$, corresponding to a mean pressure $P = 25 \times 10^{-5}$ atm.;[24] hence log $p =$ log $S - 12.6$.

Values for sun-spots on the same scale can be found from Miss Moore's data.[25] The data for H, C, N, and O depend on lines of high excitation potential, and are unfortunately somewhat uncertain.

[23] H. N. Russell, *Mt. Wilson Contr.*, No. 383, Table XIV; *Astrophysical Journal*, **70**, 56, 1929.

[24] *Op. cit.*, p. 44.

[25] C. E. Moore, *Mt. Wilson Contr.*, No. 446, Tables XIII, XIV; *Astrophysical Journal*, **75**, 222, 1932.

Table II gives the adopted values for neutral atoms (the ions taking no part in the reactions). The increase in the spots arises partly from lower opacity, but mainly from diminished ionization as T drops from 5740° to 4720°. The values for C and N depend mainly on infra-red lines observed by Babcock;[26] for H, on a compromise between the writer's earlier and later estimates (see below); for B and F, they are guesses based on the general course of the plot of log S against atomic number. The values of S for the disk have been

TABLE II

VALUES OF LOG S

El.	Disk	Spot	El.	Disk	Spot	El.	Disk	Spot
H.....	10.5	10.7	*F*.....	8:	8.2:	*Ti*.....	3.6	4.9
B.....	5:	5.5:	*Mg*.....	7.0	8.0	*Cu*.....	4.3	4.9
C.....	7.5	7.7	*Al*.....	4.6	5.9	*Zn*.....	4.9	5.2
N.....	8.0	8.2	*Si*.....	7.0	7.5	*Zr*.....	0.9	2.1
O.....	9.0	9.2	*Ca*.....	4.6	5.9			

determined directly from atomic lines and should be independent of the amount of compounds present.

We have then

$$\log S_{AB} = \log S_A + \log S_B - \log K_A - 12.6 . \qquad (5)$$

The results, which are given in Table III, show that only a small fraction of the atoms are tied up in compounds. Above the spots, however, the abundance of CO is comparable with that of C, and $S(C)+S(CO)$ must be equated to the tabular value for carbon.

6. The observed values of log S for the molecular spectra that have been identified in the disk or in spots[27] are added for comparison. Since the faint band lines are often blended in the spot spectrum and only rough estimates are possible, log S has been taken as 3, 2, 1, or 0 according as the lines are conspicuous, medium, weak, or barely visible. The letter a denotes that the bands are definitely

[26] Unpublished.

[27] R. S. Richardson, *Mt. Wilson Contr.*, No. 422; *Astrophysical Journal*, **73**, 216, 1931 (also personal communication). Pearse, *Publications of the American Astronomical Society*, **7**, 12, 1931.

absent; a dash, that they are out of reach in the ultra-violet; and a blank, the absence of data.

The computed and observed values are both so rough that little more than a qualitative test is possible; but this is satisfactory. The prominence of hydrides is independent proof of the great abundance of hydrogen; but the bands average weaker than calculated, suggesting that the assumed abundance of *H* may be too great. The

TABLE III

MOLECULES IN THE SUN

	Disk		Spot			Disk		Spot	
	Comp.	Obs.	Comp.	Obs.		Comp.	Obs.	Comp.	Obs.
H_2	6.0	*a*	7.4	?	O_2	2.9	–	4.4	–
BH	−0.3	.....	2.1	.....	*BO*	0.2	*a*	2.3	1.4
CH	3.1	3.0	4.1	3	*CO*	5.1	–	7.3	–
NH	4.0	2.1	5.3	–	*NO*	3.4	–	5.0	–
OH	5.3	3.0	6.7	–	*AlO*	−1.9	*a*	0.4	1
MgH	1.4	1	2.9	3	*SiO*	3.1	–	5.3	–
AlH	−0.7	*a*	1.4	1	*TiO*	−0.6	*a*	2.2	3
SiH	2.2	1	3.5	2	*ZrO*	−2.4	*a*	0.3	0:
CaH	−0.4	*a*	1.5	3	C_2	0.9	1.3	2.4	2
CuH	−0.7	.....	0.7	.....	*CN*	2.3	3.2	3.9	3
ZnH	−1.3	.....	−0.5	.....	N_2	3.4	–	5.4	–
SiF	0.9	1.5:	2.6	2	*SiN*	1.3	*a*	3.2	.....

writer's recent study of maxima of stellar lines[28] led to the higher value log $S = 11.0$; but the uncertainty of this determination is fully 0.5. When the calibration of the solar lines has been improved and the dissociation potentials are better determined, the hydrides may give the best value for the solar abundance of hydrogen.

The large discordance for *OH* may come partly from difficulties in the calibration of faint lines near λ 3000. The fair agreement for *BO* and *SiF* shows that the estimates of abundance for *B* and *F* are of the right order of magnitude.

[28] H. N. Russell, *Mt. Wilson Contr.*, No. 477; *Astrophysical Journal*, **78**, 239, 1933.

If Δ denotes the change from disk to spot,

$$\Delta \log S_{AB} = \Delta \log S_A + \Delta \log S_B - \Delta \log K - \frac{E}{5040} \Delta \left(\frac{1}{T}\right), \quad (6)$$

where E is the excitation potential of the molecule in its absorbing state. For all bands so far observed $E=0$; $\Delta \log K$ is nearly $-0.2D$; ΔS is 0.2 for the non-metals, which are not sensibly ionized, but is 1.3 for *Ca* and *Ti*. The disappearance of the band spectra of metallic compounds in the disk is therefore due as much to ionization of the atoms as to dissociation of the molecules. Both effects are at maximum for *TiO*.

The most abundant molecule is H_2; but its strongest observable lines should come from a $^3\Pi$ level with $E=11.76$. For the number of molecules in this state we find $\log M = -3.3$ (disk) and -4.2 (spot). Richardson's conclusion that these lines are not present is thus fully confirmed; indeed, they should be *weakened* in the spots, like other lines with large E.

7. On the same principles, we may calculate the amounts of various compounds to be expected in giant and dwarf stars of different temperatures, using the results recently derived by the writer.[29] These figures show that below $T=6300°$ (or $z'=5040/T=0.8$) the number, N, of atoms above unit area of the photosphere varies almost linearly with z—with minor fluctuations due to the successive ionization of the elements assumed to be present. The fluctuations must be smaller in the actual, more complex atmosphere, and to neglect them is desirable, so that our diagram may show the uncomplicated effects of dissociation. The effect of compounds on the general opacity, which must perforce be neglected in our present ignorance, may well be greater.

In the arbitrary units of the diagram

$$\log N = 5.00 + 0.35z' \text{ (giants)},$$
$$= 5.20 - 0.45z' \text{ (dwarfs)}.$$

For the sun (dwarf with $z'=0.88$), $N=4.80$. If, then, T is the total number of atoms of a given kind (in all states of ionization or com-

[29] *Ibid.*, Fig. 1, p. 284; Fig. 2, p. 286.

bination) above unit area in a star and T_0 the value for the sun, we have, on the assumption of similar composition,

$$\left.\begin{aligned}\log T &= \log T_0 + 0.20 + 0.35z' \text{ (giants)} ,\\ &= \log T_0 + 0.40 - 0.45z' \text{ (dwarfs)} .\end{aligned}\right\} \tag{7}$$

The surface gravity is given by the empirical relations,[30]

$$\log g = 0.76 - 2.23z' \text{ (giants)} = -0.65 + 0.64z' \text{ (dwarfs)} . \tag{8}$$

The total effective pressure (assumed to be half that at the bottom) varies as Ng.

Now let Ny be the actual number of particles in the atmosphere (so that y measures the mean degree of association into molecules) and let p' be the pressure which would be exerted by T free particles. Then $p' = Tgy/C$. For the sun, $y = 1$, $\log p = \log T - 12.6$, whence $\log C = 12.52$,

$$\left.\begin{aligned}\log p' &= \log T_0 - 11.56 - 1.88z' + \log y \text{ (giants)} ,\\ &= \log T_0 - 12.77 + 0.19z' + \log y \text{ (dwarfs)} .\end{aligned}\right\} \tag{9}$$

With hydrogen in great excess, only the formation of H_2 affects y sensibly. If x is the fraction of the molecules dissociated, $y = 2/(1+x)$. For any given element A, we then have

$$\left.\begin{aligned}p'(A) &= p(A) + 2p(A_2) + p(AB) + p(AC) + \cdots ,\\ p'(A) &= p(A)\left[1 + \frac{2p(A)}{K(A_2)} + \frac{p(B)}{K(AB)} + \cdots\right] .\end{aligned}\right\} \tag{10}$$

For a given temperature we thus obtain a set of simultaneous quadratic equations for $p(A)$, $p(B)$, ... The solution is usually easy. Hydrogen is so abundant that by treating it alone we obtain sufficient approximations for $p(H)$ and y. The other elements fall into groups which have little mutual influence; indeed, each may usually be handled separately. Finally,

$$S(AB) = \frac{Cp(AB)}{gy} .$$

[30] *Ibid.*, p. 270 (noting that $z' = 2.30z$).

The abundance of non-metallic atoms and molecules is thus found. For the metals, ionization must be included. If $K'(A)$ is the usual ionization constant, P the electron pressure, and p' the pressure of the ions, $Pp'(A) = K'(A)p(A)$, and the term $K'(A)/P$ must be added within the brackets in (10). For P we may use the linear formulae

$$\log P = -2.55 - 2.74z' \text{ (giants)},^{31}$$
$$= -3.23 - 1.36z' \text{ (dwarfs)}.$$

8. The results for giant stars are shown in Figure 1. The principal compounds having observable bands are indicated by heavy lines, other observable substances by dashed or dotted lines, and those whose strong lines are inaccessible by thin lines. Until compounds form in considerable quantity, the curve for an element is practically straight, rising slowly to the left, according to (7). As combination occurs, the curve for its principal compound rises, crosses the other, and continues along the same asymptote, as for *C* and *CO*. The more firmly bound the compound, the steeper are the sloping portions of the curves, which again are nearly straight. The formation of *CO* has very little effect on *O*, because oxygen is in great excess.

The principal "compound" of *N* is N_2, for which the flat part of the curve is lower by 0.3 since the number of free particles is halved.

The curves for Ti^+ at the right and *TiO* at the left have the same asymptote. That for *Ti* almost reaches the line, but not quite, since the amounts of the other phases are sensible. The moderate slope of the *TiO* curve near $z' = 1.5$ shows the effect of dissociation alone, the steep slope at $z' = 1$ that of dissociation and ionization combined. The curves for *CH*, *CN*, and C_2 have maxima. At high temperatures, where the percentage of compounds is small, they rise in the usual way in cooler stars; but when *CO* begins to form in quantity, the resulting depletion of the free carbon forces them down. At low temperatures this firmly bound compound takes more and more of the carbon away from the others; *CN*, which is tighter bound than *CH*, shows a smaller effect, and would show still less were it not for the locking-up of *N* in N_2; while C_2 exhibits a double effect. Toward the right the curve for *CH* rises above the others (because *H* is so

[31] Russell (*ibid.*, p. 285), changing dynes cm^{-2} into atmospheres.

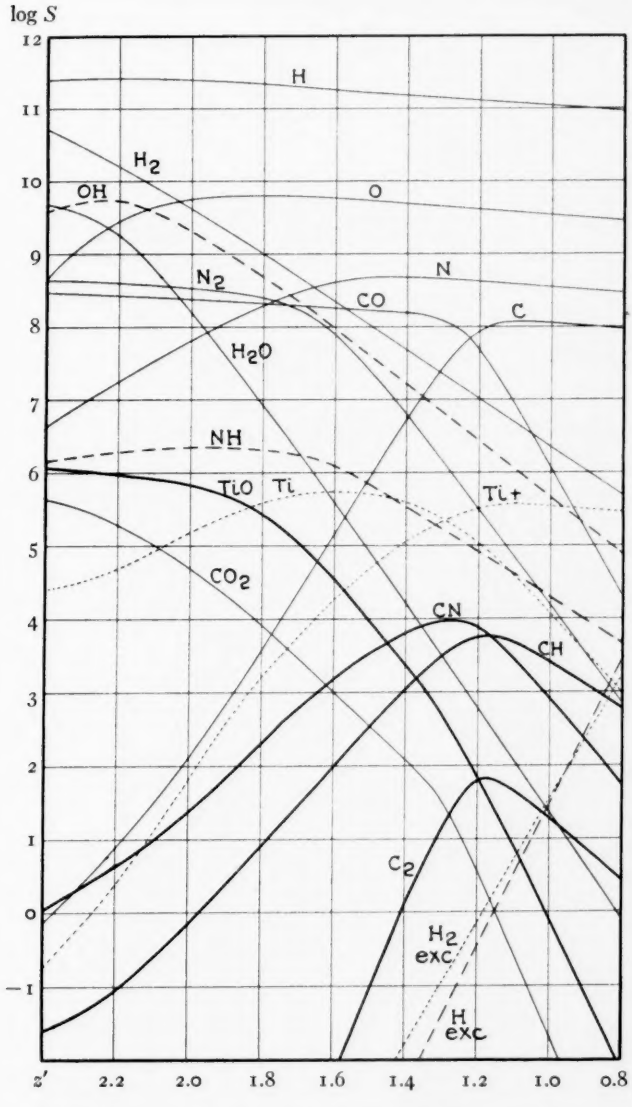

FIG. 1.—Abundance of molecules and atoms in giant oxygen stars (K–M). The ordinates are logarithms of the numbers of atoms or molecules per unit area; the abscissae, the values of $z' = \frac{5040}{T}$.

abundant) and gains more and more with rising temperatures, because CH is loosely bound. The relative height of the maxima for CH and CN depends on the assumed abundance of N compared with H, which is uncertain.

At these low densities the formation of H_2O is important only at the lowest temperatures (depressing the curve for OH), and that of CO_2 is insignificant.

The bands of NH and OH should be very strong in the cooler stars, but the spectra of such bodies have not yet been followed far enough into the ultra-violet. The sharply ascending lines in the lower right-hand corner represent the number of excited atoms of H (Balmer series) and a million times that of the excited molecules of H_2. There is no hope of observing the latter in any stellar spectrum. They should be strongest in the white dwarfs, with their high atmospheric pressures. Even then the calculated number is too small.

9. The appearance of low-temperature metallic lines in emission in long-period variables may be connected with the presence of band absorption. For example, the blue-green bands of MgH represent a transition upward into a molecular state which dissociates into a normal H atom and an excited Mg atom in the ^3P-state. If the dissociation occurs, the latter, in returning to normal, will emit λ 4571, one of the most conspicuous of the emission lines[32] found in the stars. The apparently capricious appearance of bright iron lines of some multiplets and not of others may perhaps be explicable by a similar relation, but no band spectra of iron compounds have yet been analyzed.

The conspicuous weakening of the lines of titanium at minimum is in accordance with Figure 1. The lines of vanadium persist, as do those of aluminum and potassium,[33] while the D lines of sodium are surprisingly inconspicuous in types M6e–M8e. Merrill[34] suggests that the supply of free sodium atoms is exhausted by the formation of compounds. For the oxide of any element X we have from (1)

$$\frac{p(XO)}{p(X)} = \frac{p(OH)}{p(H)} \frac{K(OH)}{K(XO)}.$$

[32] A. H. Joy, *Mt. Wilson Contr.*, No. 311; *Astrophysical Journal*, **63**, 281, 1926.

[33] *Ibid.*, p. 294.

[34] P. W. Merrill, *Mt. Wilson Contr.*, No. 306; *Astrophysical Journal*, **63**, 15, 17, 1926.

Now (4) may be written $\log K = A + \frac{1}{2} \log T - Dz'$, whence

$$\log \left(\frac{p(XO)}{p(X)} \right) = \log \left(\frac{p(OH)}{p(H)} \right) + A(OH) - A(XO) + z'[D(XO) - D(OH)] .$$

With the assumed proportions of oxygen and hydrogen, $\log [p(OH)/p(H)]$ never exceeds -1.6, either for giants or for dwarfs (Figs. 1 and 2), while $A(OH) - A(XO)$ is usually small and negative (Table I); hence, approximately,

$$\log \frac{p(XO)}{p(X)} = -2 + z' \, (D - 5) .$$

If the strength of metallic lines is seriously depleted by formation of oxide, we may set $p(XO) > 3p(X)$, whence $D - 5 > 2.5/z'$ or, roughly, $D > 6$. The oxide bands may, however, be conspicuous for considerably smaller values of D.

For *AlO*, $D = 4.1$, and for *VO*, about 5 (a rough estimate by the Birge-Sponer method). The persistence of these lines is thus explained.

For *NaO*, Professor Pauling[35] writes:

I find that

$$Na + Cl \rightarrow NaCl(g) + 4.02 \text{ v.e.}$$
$$Na + F \rightarrow NaF(g) \; + 5.40 \text{ v.e.}$$

I believe that in this regard oxygen would be about midway between *F* and *Cl*, inasmuch as its univalent ionic radius is only slightly less than for Cl^- and its electron affinity probably approaches that of *F*. Hence I suggest

$$Na + O = NaO + 4.7 \pm 0.5 \text{ v.e.}$$

It follows that no large fraction of the sodium atoms can be locked up as *NaO* in any stars for which hydrogen is in large excess over oxygen. Since fluorine is considerably less abundant than oxygen, the same will be true of *NaF*.

The weakening of the sodium lines can therefore hardly be attributed to this cause. A later suggestion of Merrill's[36] that the lines are obscured in some way by the heavy overlying bands of *TiO* is to be preferred.

[35] Personal communication.

[36] P. W. Merrill and C. G. Burwell, *Mt. Wilson Contr.*, No. 399; *Astrophysical Journal*, **71**, 330, 1930.

10. Figure 2 shows similar curves for dwarf stars. It closely resembles Figure 1, but the maxima for the carbon compounds come at higher temperatures, on account of the greater pressure, as does also

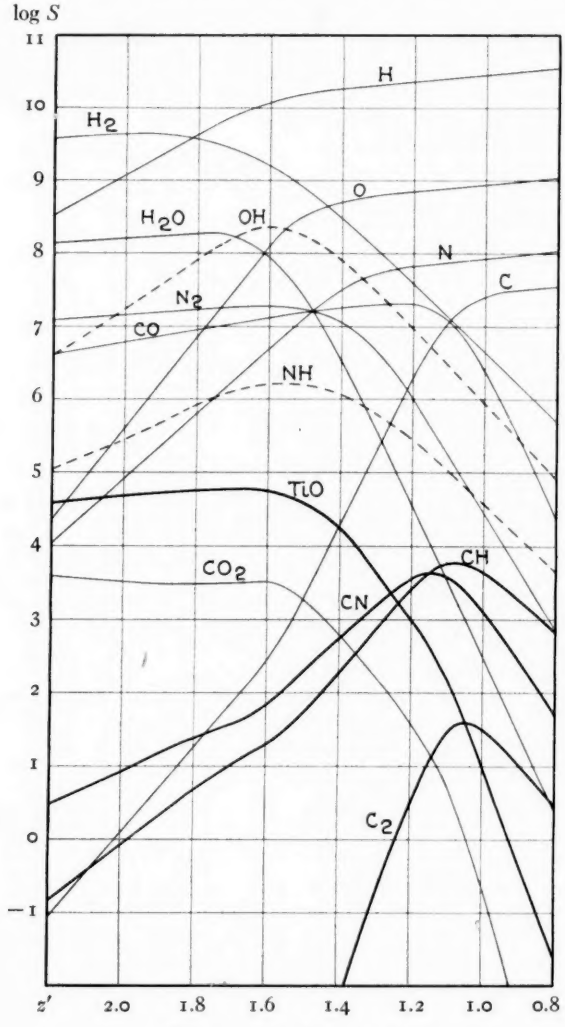

FIG. 2.—Dwarf oxygen stars

the full development of *TiO*. The heights of the maxima are lower than in giants because of the greater opacity of the denser atmosphere.

The dissociation potentials for the removal of the two hydrogen atoms from H_2O are nearly equal, so that the curves for O, OH, and H_2O cut one another close together. For CO_2 the removal of the first oxygen atom requires 6.3 volts and of the second 10 volts, and the curves are widely separated. If the oxygen were not used up in forming OH and H_2O, the CO_2 curve would continue to rise to the left of $z' = 1.6$ and approach that for CO. The latter is not sensibly affected, since practically all the carbon is in this compound. This depletion of oxygen lowers the curve for CO_2, but raises that for C, since the product of the partial pressures of C and O is constant. This entails a change in the shapes of the CH and CN curves near $z' = 1.6$.

11. These two sets of curves depend on laboratory data relating to elements and compounds, on the calibrated intensities of solar lines, and on certain general stellar data (gravity and temperature); but no use whatever has been made even of the existence of bands in stellar spectra. The curves, therefore, represent pure theoretical prediction, and it is of great interest to compare them with observation.

Qualitatively, the agreement is excellent. The bands of CH and CN reach maxima, and fall off in the cooler stars; those of C_2 are very faint and observable only with the highest dispersion (in the sun). The TiO bands strengthen as the others fade. At intermediate temperatures (3000°–3500°) they should be stronger in dwarfs than in giants of the same temperature; that is, giants with equally strong bands should be cooler. The saturation point should come at a higher temperature in dwarfs; but the maximum intensity in giants, though found in cooler stars, should be much greater. This prediction explains the previously puzzling observation that no dwarfs have been found "later" (i.e., with stronger bands) than M5, while the giants run on to M9 on the same scale, and also the fact that the difference of color index between giants and dwarfs, which increases from F5 to K, diminishes again in class M.

Quantitatively, the values of z' may be converted into spectral classes by the accompanying table.[37]

z'	0.8	1.0	1.2	1.4	1.6	1.8
Spectrum, giants	F4	G1	G8	K4	Mo	M6
Spectrum, dwarfs	F7	G8	K6	K9	M3?	

[37] Russell, *Mt. Wilson Contr.*, No. 477; *Astrophysical Journal*, **78**, 270, 271, 290, 1933; Russell, Dugan, and Stewart, *Astronomy*, **1**, 753, 1926.

We then find

	Giants		Dwarfs	
	z'	Sp.	z'	Sp.
C_2, maximum	1.17	G7	1.05	K1
CH, maximum	1.16	G7	1.08	K2
CN, maximum	1.27	K1	1.15	K4
TiO, full intensity	2.0	M10?	1.6	M3?
equal to maximum of *CN*	1.50	K7	1.30	K8
equal to maximum of *CH*	1.45	K6	1.32	K8

The observed data on the bands of carbon compounds are rather weak. Miss Payne[38] gives the maximum for *CH* at class Ko and Lindblad[39] for *CN* at G5 to K5—both for giant stars. For dwarfs Lindblad's measures[40] show a flat maximum at Ko with intensity dropping very little at higher temperatures. His observations were made with small dispersion. Swings and Struve[41] have shown that at higher temperatures the observed "band" arises mainly from atomic lines, and that the actual bands both of *CH*(λ 4308) and *CN* (λ 4216) disappear above class F8. The C_2 bands are so faint that they have been recorded only in the sun ($z' = 0.88$). They strengthen moderately in sun-spots ($z' = 1.07$) as predicted.

Further observations of all these bands with high dispersion are desirable.

The predicted spectral classes in which the *TiO* bands should equal those of *CO* and *CH* are in fair agreement with observations, though probably a little "late." From this point to the maximum, log *S* increases by 1.1 for dwarfs and 2.0 for giants, corresponding to 2 and 4 units of Rowland's scale for isolated lines of moderate intensity.[42] The gain in conspicuousness of the bands is certainly far greater. In explanation, one may note that, in passing from the spectrum of the sun's disk to that of spots, the stronger lines of elements such as *Ca*, *Ti*, *V*, and *Cr* are intensified to about the degree calculated from the change in ionization and the calibration of Rowland's tables,[43] while

[38] *Stellar Atmospheres*, p. 194, Cambridge, 1925.

[39] *Mt. Wilson Contr.*, No. 228; *Astrophysical Journal*, **55**, 85, 1922.

[40] *Ibid.*, p. 113. [41] *Loc. cit.*

[42] Adams, Russell, and Moore, *Mt. Wilson Contr.*, No. 358; *Astrophysical Journal*, **68**, 1, 1928.

[43] C. E. Moore, *Mt. Wilson Contr.*, No. 446; *Astrophysical Journal*, **75**, 353, 1932.

the change for the fainter lines is much greater. A similar effect for the multitudes of faint lines which compose the bands would go far to account for the facts.

The *CN* bands are sensitive to absolute magnitude. If compared at corresponding temperatures, the predicted abundance of *CN* is slightly less in giants than in dwarfs above 4500°, and considerably greater at low temperatures; but, if compared for the same spectral class (as is usual in absolute-magnitude determinations), log *S* is greater in giants by 0.69 in class Go, 0.80 in G5, and 0.76 in Ko—corresponding to a conspicuous strengthening. For classes K5 and M the predicted effect is smaller. The predicted differences for *CH* in the same classes are +0.24, +0.28, −0.04, so that the G band should vary little with absolute magnitude, as Swings and Struve[44] found to be the case. All these phenomena should be more conspicuous in super-giants.

The agreement between theory and observation is thus very satisfactory for the oxygen stars—as the K–M branch of the sequence should properly be called.

Stars of class S form a side branch of the oxygen sequence. Merrill has found[45] that the bands of *ZrO* and *TiO* coexist in many stars, and that in long-period variables of this type the latter are fainter toward maximum, indicating that the temperature range for the production of the zirconium bands is higher than that for the titanium bands. This behavior suggests that the dissociation potential is higher for *ZrO* than for *TiO*. The determination from the band spectra, though very rough, confirms the conclusion but indicates that the difference in D is not great. Unless it is, zirconium must be abnormally abundant in the S stars to bring the bands to the observed intensity.

12. For the carbon stars, we may interchange the assumed abundance of *C* and *O*, and assume equations (7) and (9) to hold good. We thus obtain Figure 3 for giants. Here the curves for *CH*, *CN*, and C_2 rise steadily—the three successively predominating. Cyanogen reaches its full amount at about 2500°, while the others are still increasing at 2000°. The amounts of all three are much

[44] *Op. cit.*

[45] *Mt. Wilson Contr.*, No. 325; *Astrophysical Journal*, **65**, 23, 1927.

greater than that of *TiO* in the oxygen stars. The amounts of *TiO*, H_2O, and CO_2 are negligible.

The explanation of the splitting of the spectral sequence by chemical differences suggested years ago by R. H. Curtiss[46] is thus

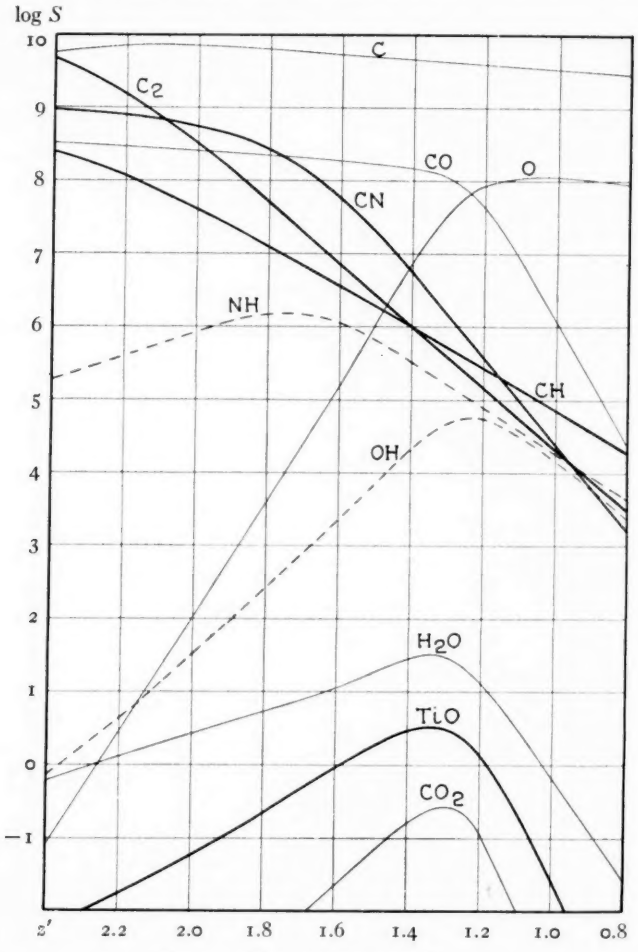

FIG. 3.—Giant carbon stars (R−N)

fully confirmed. A close quantitative agreement can hardly be expected, since the prediction depends on the arbitrary assumption that the diameters, etc., of N stars are the same as for M stars of the

[46] See also C. D. Shane, *Lick Observatory Bulletin*, **10**, 86, 1919.

same effective temperature. Moreover, the abundance of nitrogen has been assumed to be the same in both, despite the great difference for carbon.

According to Shane[47] and Rufus,[48] the R and N stars form a single sequence if arranged in order of faintness in the more refrangible region; but the strength of the C_2 bands increases rapidly from Ro to R6, drops greatly at No, and increases again to N6. The *CN* bands increase rapidly from Ro to R_5 and then weaken and are very faint in the N's. The G band of *CH* increases from Ro to R_3 and then decreases slowly and is almost absent in the later N's. The different intensity of the *CN* bands in classes No and Ro (seen clearly on Shane's Pl. XI) may reasonably be attributed to differences in the abundance of nitrogen. The *CN* bands in the violet disappear in the later N stars, whose spectra can be extended beyond the blue only by very heavy exposures. The band at λ 4607, which belongs to the same band system, increases from Ro to R8 and then remains nearly constant from No to N_5.

The red cyanogen bands are so strong in the N stars that the occasional gaps between the band lines resemble emission lines. The bands do not increase in intensity in the later subdivisions of the class.[49]

This system and the λ 4607 band of the violet system behave similarly. These systems are both absorbed by the *CN* molecule in its normal state. It appears probable that the disappearance of the more refrangible bands of the latter arises from the fading-out of the continuous background (which is excessively weak in the later N's) rather than from absence of absorption. The disappearance of the *CH* band may be similarly explained. Further observations are desirable, but would demand very long exposures.

Otherwise the agreement of theory and observation is good. The steady gain of the C_2 bands in the reddest stars, while the *CN* bands are stationary, and the relative weakening of *CH* are clearly indicated in Figure 3. The maximum strength of the bands much exceeds that of the *TiO* bands in M stars, for the gaps in the latter,

[47] *Ibid.*, **13**, 123, 1928.

[48] *Publications of the Observatory of the University of Michigan*, **2**, 140, 1916.

[49] R. F. Sanford, personal communication.

even in extreme cases, do not appear as spurious bright lines. The absolute increase in the *CH* bands in the redder stars appears to be much smaller than predicted. There is a possibility that the unidentified bands discovered by Sanford[50] in the late N's may arise from some compound, whose formation uses up the *CH*, but this is speculative at present.

The appearance with considerable strength of the C_2^{13} isotope band in the later N stars, although it is excessively faint in the laboratory,[51] may arise from the strengthening of faint lines and bands relative to heavy ones already mentioned. It is worth remarking, however, that the general presumption that isotopes will be found in about the same abundance everywhere is hardly safe to apply to stars in which it is certain that the relative abundance of the elements, at least of carbon and oxygen, is quite unusual.

13. Figure 4 shows similar curves for dwarf carbon stars of the same assumed composition. Here C_2, *CH*, and *CN* are about equally abundant. At 4000° ($z' = 1.26$) these bands should be stronger than the *TiO* bands in any dwarf. Dwarf stars of this temperature would be reasonably bright, and stand a good chance of being observed. The complete absence of R and N stars among known dwarfs can be explained only if they are very infrequent compared with the M's. This is certainly true among the giants. Brighter than the eighth visual magnitude there are 1312 stars[52] of class M, 6 of class R, and 32 of class N (including 20 variables whose median magnitude is brighter than the limit).[53]

The number of red dwarf stars at present known is about one hundred and twenty.[54] If the observational selection were the same for giants and dwarfs, one might then expect to find two or three carbon stars among the dwarfs. The great brightness of the giant N's, however, gives them an advantage, while dwarf N's, owing to their low temperature, would be very faint and at a disadvantage.

[50] *Publications of the Astronomical Society of the Pacific*, **38**, 177, 1926.

[51] *Ibid.*, **41**, 271, 1929; D. H. Menzel, *ibid.*, **42**, 34, 1930.

[52] H. Shapley and A. J. Cannon, *Harvard Circular*, No. 245, 1923.

[53] From a manuscript list of R and N stars prepared by Mr. Shane and lent by Mr. Sanford.

[54] From a manuscript list by Mr. van Maanen.

It is not surprising, therefore, that none has been found, but the discovery of one or two would be no more surprising.

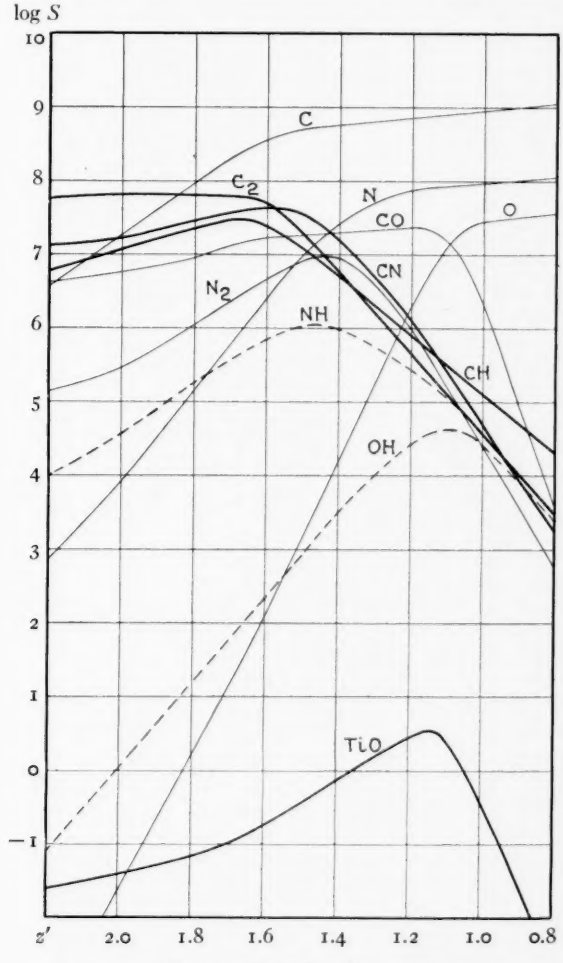

FIG. 4.—Dwarf carbon stars

The case in which carbon and oxygen are exactly balanced is represented in Figure 5—for giant stars in which log $T = 9.0$ for both. The carbon bands should be strong, though weaker than in typical N's, while *TiO* would be present but much less conspicuous.

With $T > 3000°$ ($z' < 1.6$), the star would probably be classified as R pec. At $2500°$ ($z' = 2$) a 1 per cent excess of oxygen would raise

its partial pressure about fifteen fold, decreasing that of carbon in the same ratio, and give an atmosphere in which *CN* and *TiO* were both present in fairly large amount, while C_2 was very weak. Since

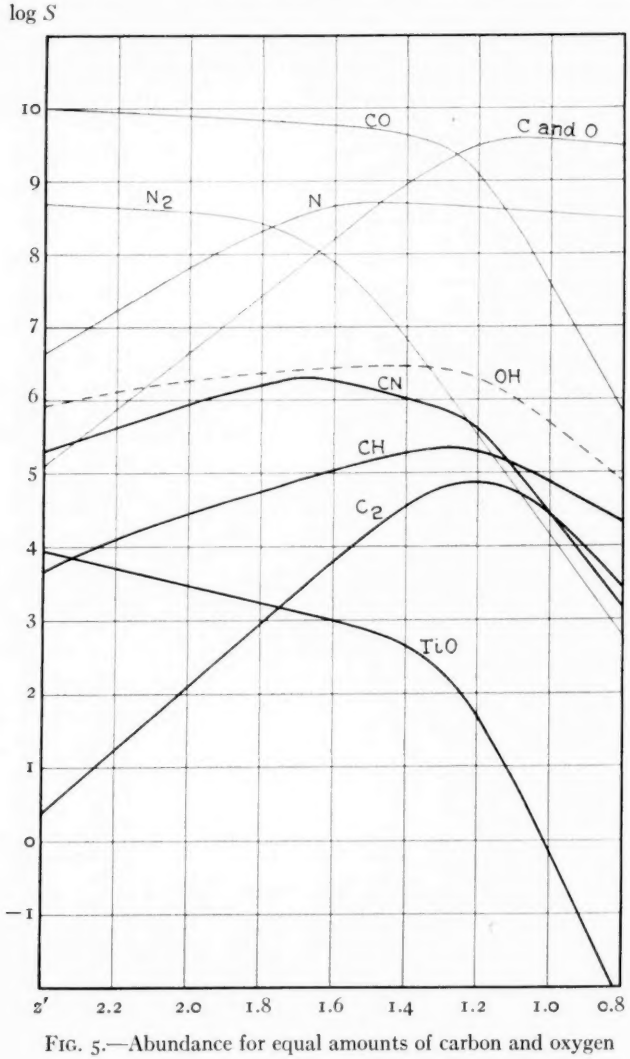

FIG. 5.—Abundance for equal amounts of carbon and oxygen

the violet *CN* bands would be unobservable at so low a temperature and the red *CN* bands would be confused with those of *TiO*, the star would then probably be described as of class M. The chance

of finding a body with so delicately balanced a composition is very small.

14. In conclusion it should be remarked that the calculations and diagrams take into account only the most firmly bound compounds. At temperatures below 2500° more complex compounds may form and the course of the curves be changed.

For example, for the reaction $HCN = H + CN$, $D = 94$ K cal $= 4.12$ volts, while for $C_2N_2 = 2CN$, $D = 3.34$ volts.[55] Since H is far more abundant than CN, the first reaction will have the advantage. If the value of K is of the same order as for other hydrides with similar D, such as CH, the ratio HCN/CN in giant stars varies from 2×10^{-5} when $z' = 1.0$ to 0.035 when $z' = 2.4$. The depletion of CN by the reactions should therefore be insignificant. The formation of C_2N_2 does still less.

In the long-period variables, especially at minimum, a great many reactions may happen, despite the very low density—even the condensation of solid or liquid particles of refractory compounds like carbides and nitrides.[56] The chemical equilibrium in such cases may be very complex, and it is not surprising that great numbers of unidentified lines are found in the spectra.

The foregoing discussion is quite definitely a reconnaissance. Its success has, however, been sufficient to lead to the hope that much work of interest may be done in the field where astronomy and chemistry overlap.

It is a pleasure to acknowledge the help which the writer has received from discussions with Messrs. Adams, Merrill, and Sanford, and especially to thank Professor Pauling and Mr. Stitt for the original data mentioned in § 4.

15. Just as the present discussion was completed, two important papers[57] arrived. The discussions of molecular dissociation in stellar atmospheres by Miss Cambresier and Dr. Rosenfeld follow the same general lines as the writer's and reach similar results.

They have assumed that all the stellar atmospheres with which they deal are composed of approximately equal numbers of non-

[55] Kistiakowsky, *loc. cit.* [56] R. Wildt, *Zeitschrift für Astrophysik*, **6**, 345, 1933.

[57] Y. Cambresier and L. Rosenfeld, *Monthly Notices of The Royal Astronomical Society*, **93**, 710, 1933; L. Rosenfeld, *ibid.*, p. 724.

ionized and ionized atoms, so that the electron pressure is one-third the total pressure. This is so far from the truth that it is interesting to see why it has not vitiated their results.

They first compute the electron pressure P_0 at the photosphere (which, according to their equations, varies as $g^{\frac{1}{2}}T^{11}$), adopting the fixed "extreme values" $g=4.9$ cm/sec.^{2} for giants and 1.9×10^5 for dwarfs.

If modified to correspond to the values of g adopted by the writer, the results compare as tabulated. The values of the electron pressure

	T	2000°	2500°	3500°	6000°
Giants:	log P_0(C and R)	−8.09	−7.28	−6.18	−4.91
	(HNR*)	−9.17	−7.78	−6.20	−4.55
	log (p/P_0) HNR	3.67	3.23	2.73	2.23
Dwarfs:	log P_0(C and R)	−5.23	−5.08	−4.86	−4.41
	HNR	−6.36	−5.68	−4.88	−4.07
	log (p/P_0) HNR	4.90	4.11	3.21	2.28

* Increasing the values of p by 0.30 to obtain the pressure at the bottom.

agree as well as could be expected, considering the very different ways of treating the opacity problem, but those of the gas pressure differ greatly, since they have assumed it to equal P_0 while the writer often makes it a thousand times as great. Their assumption of a fixed value of g increases the discordance for the hotter giants and diminishes it for the dwarfs, but leaves it very large.

The calculated temperatures for similar degrees of dissociation are therefore much lower according to these authors than in the present work; for example, they give the maximum of CN in giants at 2800°, as against 4000° found above.

Qualitatively, however, the agreement of the two sets of results is good, confirming their conclusion that the general character of the results is quite insensitive to wide changes in the initial assumptions. Such details as the effect of ionization upon the metallic compounds in sun-spots, however, could not have been revealed by so summary a treatment.

CARNEGIE INSTITUTION OF WASHINGTON
MOUNT WILSON OBSERVATORY
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HD 33232, A LONG-PERIOD SPECTROSCOPIC BINARY OF CLASS Be[1]

By PAUL W. MERRILL

ABSTRACT

The spectrum of HD 33232 combines features of types nB3 and cA0. *Hβ* has an emission fringe which shifts from the red to the violet edge as the position of the absorption line changes.

Displacements of lines of *H*, *Fe* II, *Mg* II, *Ca* II, *Si* II and III, are in Table I. The data for *hydrogen* give a well-defined curve of variation from which the following orbital elements were computed: $P=3710$ days, $e=0.31$, $K_1=31.5$ km/sec., $a_1 \sin i=1{,}528{,}000{,}000$ km, $m_2^3 \sin^3 i/(m_1+m_2)^2=10.4\odot$, indicating, if the two components are equally massive, a minimum systemic mass eighty-three times that of the sun.

Iron, *silicon*, and *magnesium* lines exhibit greater amplitudes than hydrogen lines; *calcium* and *helium* lines, smaller amplitudes. These phenomena are difficult to explain.

Comparisons with other Be spectra are made in a brief discussion.

A bright *Hα* line was discovered in the spectrum of HD 33232[2] on an objective-prism photograph taken by Mr. M. L. Humason on December 29, 1920.[3] The first slit spectrograms, taken as a regular part of the observing program on Be stars,[4] showed *Hβ*, *Hγ*, and *Hδ* to consist of absorption lines with emission on their violet edges. The unusual structure of these lines led to the investigation reported here.

The spectrum appears to combine features ordinarily found in different types. The helium lines are broad, ill defined, and of low intensity. *Mg* λ 4481 is somewhat weaker and narrower than *He* λ 4471. The intense and sharply defined absorption lines of hydrogen suggest the c characteristic, as does also the presence of narrow lines of ionized iron and silicon, although these metallic lines are of low intensity. Except for the strength of the hydrogen lines and possibly the low intensity of λ 4481, the spectrum might be described as a result of superposing types nB3 and cA0.

The measured displacements of lines of hydrogen, iron, magnesium, calcium, and silicon are in Table I. The values in parentheses have low weight.

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 491.

[2] BD +40°1196; R.A. $5^h3^m.7$, Dec. +40°53′, 1900; mag. 8.1; spectrum (*HD*) B3.

[3] *Publications of the Astronomical Society of the Pacific*, **33**, 112, 1921.

[4] *Mt. Wilson Contr.*, No. 294; *Astrophysical Journal*, **61**, 389, 1925.

TABLE I

DISPLACEMENTS OF LINES OF VARIOUS ELEMENTS
(Individual Plates)

Plate	Date	J.D. 2420000	Log *Ev/Er*	*H*	*Fe* II	*Mg* II	*Ca* II	*Si* II, III
				km/sec.	km/sec.	km/sec.	km/sec.	km/sec.
C 848..	1921 Jan. 4	+2694	0.66	+47.1	+74.7	.....	(+22.0)	.....
856..	Jan. 5	2695	0.70	+49.1	+90.6	+76.7	.....	+82.8
922..	Feb. 27	2748	0.62	+48.4	+85.5	+91.6	.....	+74.2
1264..	Aug. 25	2927	0.52	+45.3	+62.3	+74.7	.....	+66.4
1520..	1922 Jan. 12	3067	0.31	+34.4	+32.6	.....	.....	.....
1624..	Mar. 19	3133	0.30	+37.0	+34.2	(− 6.3)	.....	.....
1960..	Oct. 10	3338	0.31	+26.1	+13.2	−13.0	.....	(+ 5.2)
2417..	1923 Aug. 31	3663	9.75	− 0.3	−26.2	−17.7	(+ 1.2)	(+ 4.6)
2461..	Sept. 28	3691	9.66	− 2.1	−33.2	.....	−30.6	.....
2473..	Oct. 1	3694	9.77	+ 0.9	−28.6	(+16.5)	.....	.....
2591..	Dec. 24	3778	9.37	− 8.0	−43.0	−27.5	.....	.....
2725..	1924 Mar. 24	3869	9.46	−23.7	−38.0	−40.5	.....	.....
3006..	Sept. 13	4042	9.35	−17.9	(−33.6)	.....	−15.9	.....
3190..	1925 Mar. 4	4214	9.62	−18.6	.....	.....	−39.2	.....
3543..	Oct. 1	4425	9.51	−16.8	.....	.....	+ 9.8	.....
3701..	1926 Feb. 22	4569	9.60	−17.2	.....	.....	.....	.....
3936..	Aug. 12	4740	9.79	− 4.3	(− 1.4)	(+22.1)	(− 6.3)	.....
4127..	Dec. 13	4863	9.83	−10.2	(+10.7)	−12.9	− 7.8	.....
4424..	1927 Sept. 10	5134	9.89	+ 9.6	.....	.....	.....	.....
4480..	Oct. 13	5167	0.03	− 1.9	(+24.2)	.....	+ 1.2	.....
4586..	Dec. 22	5237	0.03	− 0.3	.....	.....	+ 3.8	.....
4709..	1928 Mar. 17	5323	9.85	+ 8.3	.....	.....	(+15.1)	.....
4960..	Aug. 29	5488	0.06	+10.8	.....	+32.4	− 1.2	.....
5060..	Nov. 1	5552	0.12	+17.0	+57.0	+21.8	+11.8	.....
5096..	Dec. 20	5601	0.18	+11.6	(+37.3)	.....	(+ 7.7)	.....
5145..	1929 Mar. 1	5672	0.08	+17.0	(+23.0)	.....	(+ 2.1)	.....
5271..	Aug. 17	5841	(0.31)	(+31.4)	.....	.....	.....	(+52.7)
5310..	Sept. 13	5868	0.44	+29.3	+57.3	+78.3	.....	.....
5353..	Nov. 20	5936	0.32	(+ 9.4)	+25.3	(+55.3)	(− 0.8)	.....
5402..	1930 Feb. 12	6020	0.49	+21.0	+42.7	(+68.4)	.....	.....
5614..	Nov. 12	6293	0.42	+37.4	+53.4	.....	.....	.....
5639..	Dec. 3	6314	0.32	+38.5	+39.8	.....	+ 5.1	.....
5856..	1931 Oct. 1	6616	0.44	+42.0	+31.6	+24.6	+16.0	.....
5877..	Oct. 27	6642	0.58	+41.2	+42.5	.....	+32.2	(+77.2)
5961..	1932 Feb. 23	6761	0.47	(+45.4)	+32.5	+13.0	.....	.....
6097..	Aug. 17	6937	0.37	+28.5	+ 5.6	.....	+ 5.8	.....
6179..	Nov. 15	7027	0.18	+33.3	.....	.....	+12.4	.....
6210..	1933 Feb. 11	7115	0.04	+ 5.2	(−18.0)	.....	(− 4.3)	.....
6231..	Mar. 6	7138	0.05	(+19.1)	(+16.8)	.....	.....	.....
6308..	Sept. 7	7323	9.86	+ 8.9	−32.0	(−22.7)	+ 3.2)	.....
6322..	Oct. 6	+7352	9.96	+ 2.6	−26.0	.....	− 0.9	−15.1

Since the hydrogen lines $H\beta$, $H\gamma$, $H\delta$, and $H\epsilon$ do not appear to differ systematically, results from all these lines have been included in the tabulated mean values. $H\gamma$ and $H\delta$, however, have usually been given double weight. The four plates taken in 1921 gave accordant velocities; but those in 1922 yielded lower values, and observations in the next few years made it clear that we were dealing with a long-period variation of considerable amplitude.

The velocities from individual plates, as well as normal places formed from groups of two to four plates (Table II), are plotted in Figure 1. Preliminary orbital elements found from charts of orbits having various combinations of e and ω, prepared from tables computed by E. S. King[5] and improved by plotting trial ephemerides, led to the elements in Table III from which the curve in Figure 1 was plotted.

All the normal places are well represented except the first three, the deviations of which suggest that the first maximum observed was higher than the second and the first descending branch of the curve less steep than that in the second cycle. The elements adopted assign, in effect, small weight to the observations at the first maximum. A compromise orbit, such as that yielded by a least-squares solution, would not, however, differ radically from that chosen.

The well-marked emission fringe on the violet edge of the absorption lines shown on the first few plates weakened as the velocity decreased. At the same time traces of emission appeared on the opposite edge and continued to develop until at minimum velocity emission was conspicuous only on the red edge. Estimates of the relative intensity of the red and violet emission fringes are in the fourth column of Table I. When no emission was visible on one edge, that on the other edge was arbitrarily assumed to be four or five times as strong. Logarithms of the ratios were used in order that symmetrically reversed behavior might be represented by a symmetrical curve. The curve strongly resembles that of the radial velocity. It crosses the 0.00 axis (equality of the two emission components) when the velocity is equal to that of the system. At these times the integrated intensity of the visible emission is decidedly less than that observed at the time the orbital velocity differs most

[5] *Harvard Annals*, **81**, 213, 1923.

from the systemic velocity; the dark lines also are somewhat less intense.

Although the lines of the metallic elements are not so accurately measurable as those of hydrogen, their displacements may be deter-

TABLE II

DISPLACEMENTS OF LINES

(Normal Places)

J.D. 2420000	Vel. km/sec.	Wt.
H		
+2766	+47.5	4
3179	+32.5	3
3683	− 0.5	3
3824	−15.8	2
4312	−17.6	4
4802	− 7.2	2
5215	+ 3.9	4
5578	+14.1	4
5925	+23.6	3
6304	+38.0	2
6655	+42.4	2.5
6982	+30.9	2
+7245	+ 7.5	3.5
Fe II		
+2716	+85.4	5
3116	+35.6	4
3683	−29.3	3
3867	−39.1	2.5
4923	+11.2	1.5
5946	+43.7	7
6673	+35.5	3
+7185	−13.2	4
Mg II		
+2790	+81.0	3
3270	−10.8	1.5
3759	−22.1	3.5
4822	− 1.2	1.5
5520	+27.1	2
5923	+70.1	2
6688	+18.8	2
+7323	−22.7	0.5
Ca II		
+2694	+22.0	0.5
3682	−20.0	1.5
4227	−15.1	3
4822	− 7.3	1.5
5209	+ 3.9	3.5
5599	+ 6.0	4.5
6524	+17.8	3
+7132	+ 5.6	5.5
Si II and III		
+2790	+74.5	3
3500	+ 4.9	1
6242	+65.0	1
+7352	−15.1	1

mined without great uncertainty, and when combined give fairly reliable normal places (Table II). The remarkable divergence of the displacements of iron lines from those of hydrogen is far outside the errors of measurement; silicon and magnesium lines on the other

hand agree fairly well with those of iron (Fig. 2). The amplitudes shown by the three metallic elements are much greater than the amplitude found for hydrogen, while the phases, especially for magnesium, seem to be earlier.

TABLE III

ORBITAL ELEMENTS OF HD 33232

$P = 3710$ days
$e = 0.31$
$\omega = 77^\circ$
T = J.D. 2423412
$V_0 = +9.3$ km/sec.
$K_1 = 31.5$ km/sec.
$a_1 \sin i = 1{,}528{,}000{,}000$ km
$\frac{m_2^3 \sin^3 i}{(m_1+m_2)^2} = 10.4\odot$

The helium lines, although diffuse and difficult, were frequently measured, but gave for the most part scattering and uncertain results. The most reliable displacements were obtained from λ 4471:

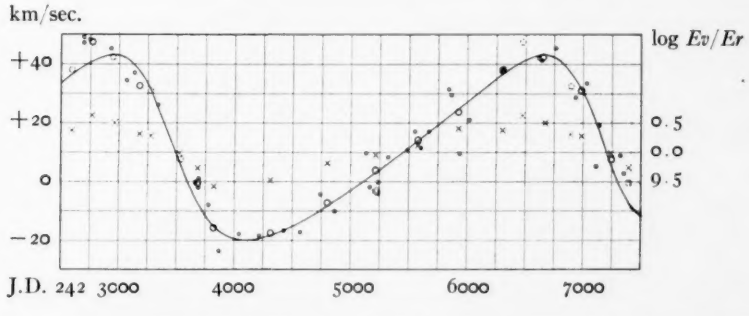

FIG. 1.—Observations of hydrogen lines. Small circles: measured displacements of dark cores; large circles, normal places. Scale at the left. Crosses: log *Ev*/*Er*, *Ev* and *Er* being the estimated intensities of the emission fringes on the violet and red edges, respectively, of the dark lines. Scale at the right.

Dotted symbols are transferred 3710 days from the observed positions.

from 10 plates entitled to the highest weight the mean value was −6.4 km/sec.; from 15 others, −5.2; mean of all, −6.0 km/sec. The individual velocities apparently have no relationship to the curve for hydrogen; moreover, the evidence is that the true displacements

either are constant or have a decidedly smaller range than those derived from the hydrogen lines.

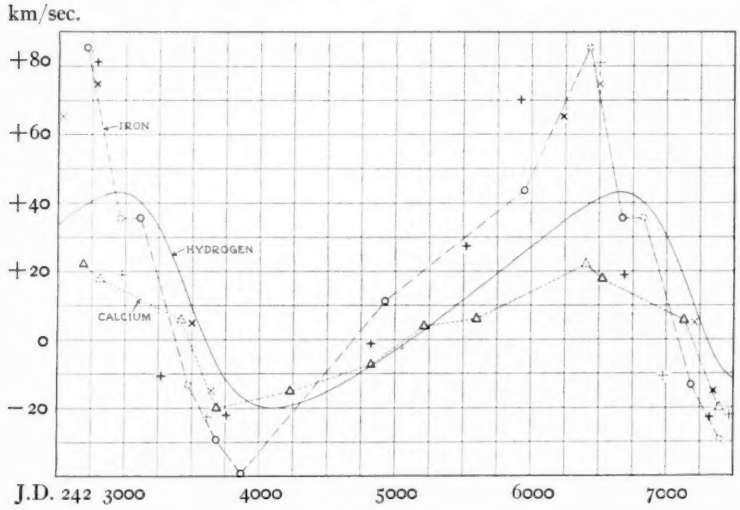

FIG. 2.—Displacements of lines of various elements.

H, smooth curve from Fig. 1.
Ca II, triangles and dotted lines.
Fe II, circles and dashed lines.
Mg II, plus signs.
Si II and III, crosses.

Dotted symbols are transferred 3710 days from the observed positions.

TABLE IV

COMPARISON OF VELOCITY-CURVES FROM CALCIUM AND HYDROGEN LINES

	Calcium	Hydrogen
Maximum	+20 km/sec.	+43 km/sec.
Minimum	−20	−20
Range	40	63
V_0	0	+ 9

The velocities derived from the calcium lines form a curve whose maximum and minimum occur at approximately the same times as those of hydrogen (Fig. 2). In amplitude and mean velocity, however, the two curves differ decidedly, as Table IV shows. If we as-

sume the lines to be blends of stationary calcium lines with stellar lines having the same displacements as the hydrogen lines, we find from the data of Table IV that the stellar lines are about 1.7 times as intense as the fixed components which must occupy a position corresponding to a velocity of −20 km/sec. This velocity differs considerably from that to be expected from interstellar lines in the spectrum of a star in the galactic position of HD 33232. The true interpretation of the observed displacements of calcium lines therefore remains in doubt.

DISCUSSION

The surprisingly long period of this star, 3710 days, is a very striking example of the tendency of Be binaries toward longer periods than those of non-emission binaries of the same type. The elements of ν Geminorum, B5e, as computed by W. E. Harper,[6] are remarkably similar to those of HD 33232, although the spectra of the two stars differ considerably.[7] The semi-major axis of the orbit of HD 33232 is extremely large, while the value of the mass function, 10.4 $\odot$, indicates, for equally massive components, a minimum systemic mass eighty-three times that of the sun. The variations in the position of the hydrogen lines in Be stars may not be due wholly to orbital motion, but a satisfactory alternative hypothesis has yet to be developed.

Variation in the structure of the hydrogen lines in Be spectra appears to be the rule rather than the exception, although in many instances observations must be extended over a term of years to disclose the changes. In a number of spectra[8] the intensities of the emission components exhibit a systematic relationship to the motion of the absorption core similar to that found for HD 33232 (p. 345). A formal explanation of this relationship follows from the assumption that a dark core moves back and forth across a somewhat wider,

[6] *Publications of the Dominion Observatory*, **4**, 279, 1919. Harper later expressed doubt concerning the validity of the elements (*Journal of the Royal Astronomical Society of Canada*, **15**, 148, 1921; see also *Publications of the Dominion Astrophysical Observatory*, **5**, 161, 1931).

[7] Other bright-line stars having notably long periods are ϕ Persei, 126.5 days; ζ Tauri, 138 days; υ Sagittarii, 138 days; BD +11°4673, 800 days.

[8] E.g., 25 Orionis and π Aquarii (D. B. McLaughlin, *Publications of the Observatory of the University of Michigan*, **4**, 37, 1931).

nearly stationary emission line, overlapping first one edge, then the other.[9] Similar motion of a hydrogen core, with respect to broad dark wings nearly fixed in position, was recently found in the spectrum of HD 31293.[10]

The widely differing amplitudes given by the displacements of lines of various elements are puzzling features. The relative behavior of iron and hydrogen lines is especially striking: the maximum velocity derived from the iron lines is nearly 40 km/sec. greater than that from the hydrogen lines, while the minimum iron velocity falls 19 km/sec. below the minimum of the orbital curve for hydrogen. The value of $a \sin i$ would thus be nearly twice as great for iron as for hydrogen, while the mass function would be seven times as great! The maximum velocities derived from the various elements differ much more than their minimum velocities. These phenomena, as well as the definite differences in the line displacements of various elements previously observed in the spectrum of BD $+11°4673$,[11] are difficult to interpret. One may well search for some explanation other than Doppler effect caused by orbital motion.

Very accurate determinations of the relative displacements of lines of different elements could readily be made in a future cycle, if desired, because the slow rate of change renders it easy to obtain numerous observations at any phase.

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[9] W. H. Christie has noted that an explanation of this type would account partially for the observed variations in the hydrogen emission of HD 198287–8 (*Mt. Wilson Contr.*, No. 475; *Astrophysical Journal*, **78**, 200, 1933).

[10] *Mt. Wilson Contr.*, No. 462; *Astrophysical Journal*, **77**, 103, 1933.

[11] *Mt. Wilson Contr.*, Nos. 381, 450; *Astrophysical Journal*, **69**, 330, 1929; **75**, 413, 1932.

THE RADIAL VELOCITY OF δ CETI

By CLIFFORD C. CRUMP

ABSTRACT

Variation in the radial velocity of 82 δ Ceti is established. The velocity of the system is approximately +12 km/sec., $P=0^d15$; $K=6$ km/sec. The elements may be slightly variable.

The variation of the radial velocity of 82 δ Ceti ($\alpha=4^h31^m$; $\delta=-0°6'$; mag. 4, B2) was discovered by Professor Edwin B. Frost and Professor Walter S. Adams from spectrograms of stars of the Orion type, made in 1901 and 1902 with the Bruce spectrograph.[1] The range of velocity variation found by them was of the order of 10 km/sec. Subsequent investigations by Dr. F. Henroteau[2] confirmed the discovery, but the early observations which he made left something to be desired. A later investigation by Dr. Henroteau[3] from spectrograms made at the Dominion Observatory in 1922 led him to announce:

> There is no doubt a good deal of uncertainty in these curves; the following conclusions, however, appear to be justified: 1. The period 0^d16122, approximately 3^h52^m, may be considered as correct. 2. The amplitude probably varies considerably. 3. There is a variation of the mean velocity as given by the individual curves. 4. There is, apparently, nothing abnormal or erratic about the radial velocity variation of δ Ceti as at first supposed. . . . .

From the time of the discovery by Frost and Adams until the investigation made by Dr. Otto Struve in 1922, seventy-six spectrograms had been obtained at the Yerkes Observatory and were measured chiefly by the writer and by Dr. Struve. Although the variation of the radial velocity seemed established, no period was found which was satisfactory. From six spectrograms made on the night of October 30, 1922, Dr. Struve, in his "Study of Spectroscopic Binaries of Short Period,"[4] found a variation of the radial velocity of 7.9 km/sec., with a mean velocity of +15.9 km/sec., and drew the conclu-

[1] *Astrophysical Journal*, **17**, 150, 1903.

[2] *Dominion Observatory Publications*, **5**, No. 11, 1922. [3] *Ibid.*, **9**, No. 1, 1925.

[4] A dissertation submitted to the Faculty of the Ogden Graduate School of Science in candidacy for the degree of Doctor of Philosophy, November, 1923.

sion that "the velocity of δ Ceti was constant, or nearly constant, on that day." It was a question as late as 1930 whether δ Ceti should be included among stars whose radial velocities vary, and it was with this in mind that the present investigation was inaugurated.

Three series of spectrograms form the basis of this study, namely, those made on the nights of November 7–8, 1930; December 2, 1931; and January 23, 1933. The entire list of spectrograms were made with the Bruce spectrograph on Eastman 40 plates. A list of the wave-lengths of the star lines used, together with the element responsible for each line, is given in Table I. The lines can be char-

TABLE I

λ	Element	λ	Element
3964.727	*He* I	4253.824	.....
3994.995	*N* II	4267.160	*C* II
4009.267	*He* I	4340.467	*H* γ
4026.189	*He* I	4387.928	*He* I
4069.788	*O* II	4414.888	*O* II
4072.156	*O* II	4416.974	.....
4075.869	*O* II	4471.477	*He* I
4101.738	*H* δ	4481.230	*Mg* II
4120.812	*He* I	4552.611	*Si* III
4143.759	*He* I	4713.143	*He* I
4168.972	*He* I	4861.326	*H* β

acterized as good; they are sharp and well defined and the spectrum is readily measurable. In measuring the displacements of the lines and in the reduction of these measurements the usual methods of procedure were employed and all the customary precautions were observed. The observed velocities were plotted and a smooth curve was drawn through them. Upon the assumption that the variation of the velocity is due to binary motion, a Lehman Filhés solution was made for each curve to determine the elements of the orbit. A list of the observations for each series, together with the number of lines measured, the length of exposure, the velocity observed, and the residulas, are given in Table II. Table III contains a list of the elements as derived from each separate determination of the orbit.

For the observations made on the night of November 7–8, 1930, a period of $0^d.1386$ seems to satisfy the observed data in a more or less adequate manner. These observations are not consecutive as are

those of the other two series, a part of the spectrograms having been made on the night of November 7 and the remainder on the night of

TABLE II

Date	Julian Day	Phase	No. of Lines	Length of Exposure	V	$V_{ob}-V_e$
	2426000+					
1930 Nov. 7–8.....	397.104	.132	9	33^m	+ 6.0	−0.3
	.132	.021	5	41	10.4	−0.2
	.167	.056	11	50	14.7	−0.9
	.197	.086	15	32	13.5	−0.4
	.226	.114	13	45	6.8	−1.2
	.316	.066	11	47	16.2	+0.1
	.356	.106	10	63	11.1	+4.3
	398.123	.042	10	61	16.8	+2.9
	.169	.087	11	64	9.6	−3.8
	.215	.134	11	63	3.8	−2.8
	.258	.138	10	57	15.0	+1.7
	2426000+					
1931 Dec. 2.....	678.022	.127	15	33	3.5	−2.6
	.046	.151	15	33	8.1	+2.3
	.073	.023	15	40	13.0	−0.9
	.101	.051	15	30	14.2	−2.7
	.124	.074	13	30	16.2	−0.7
	.169	.119	16	29	11.5	+2.5
	.192	.142	12	30	3.5	−2.6
	.215	.164	12	30	6.9	−3.6
	.238	.026	9	30	17.5	+3.0
	.262	.057	12	30	16.6	−0.4
	.285	.079	10	29	16.1	−0.5
	2427000+					
1933 Jan. 23–24.....	096.984	.124	11	16	5.4	−1.3
	.996	.136	14	14	8.4	+1.5
	097.006	.146	14	12	7.5	−1.7
	.015	.000	12	10	11.0	−1.1
	.025	.010	13	11	13.8	−1.1
	.035	.019	9	10	17.6	+0.2
	.045	.030	10	13	20.4	+2.2
	.057	.042	14	14	19.0	−1.4
	.067	.052	12	11	18.5	+2.3
	.077	.062	12	11	19.4	−1.1
	.088	.073	14	17	20.3	−0.7
	.099	.084	14	10	17.3	−0.5
	.108	.092	14	10	16.3	+0.5
	.118	.103	14	15	14.2	+1.4
	.130	.114	15	11	10.2	+1.1
	.140	.124	9	10	6.8	+0.1
	.151	.135	10	15	7.2	+0.4
	.162	.146	9	12	6.3	−3.0
	.174	.003	9	15	12.1	−1.0
	.192	.021	7	15	+19.0	−1.4

November 8. In some cases the exposure time was excessive, being as great as 1^h4^m in the case of one observation. The spectrograms are of good quality, however, and the variation of the radial velocity seems well established. The resultant curve is given in Figure 1.

TABLE III

Date	T in Julian Days	γ in Km/Sec.	K in Km/Sec.	e	ω	$a \sin i$ ×1000 Km	No. of Plates	Period Days
1930 Nov. 7–8 . .	2426397.150	+11.0	5.4	0.21	102°	10	11	0.1386
1931 Dec. 2.	2426678.212	12.4	6.9	.31	178	13	11	.1556
1933 Jan. 23–24.	2427097.145	+14.8	7.2	0.17	178	15	20	0.1556

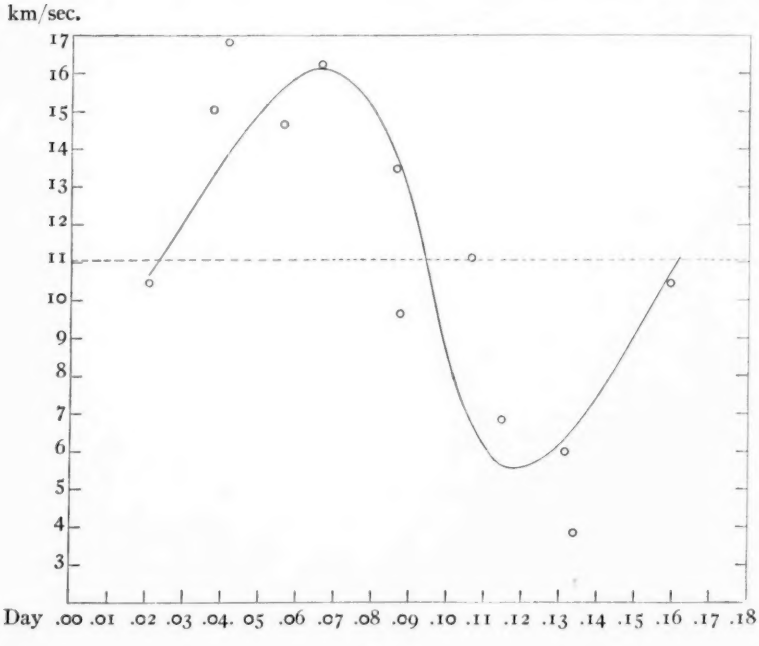

FIG. 1

The series of December 2, 1931, is better than the earlier one. A period of $0^d.1556$ satisfies the observation fairly accurately. Every element of the orbit seems to have changed from those previously determined, and the eccentricity is the greatest observed, being

0.31. Since ω is near 180°, T is uncertain, as is also the case in the later series. The velocity-curve for this determination is given in Figure 2.

The third series of observations, made on the night of January 23–24, 1933, is the best of the three. It not only contains more observations, but the exposure times are from a third to a half shorter

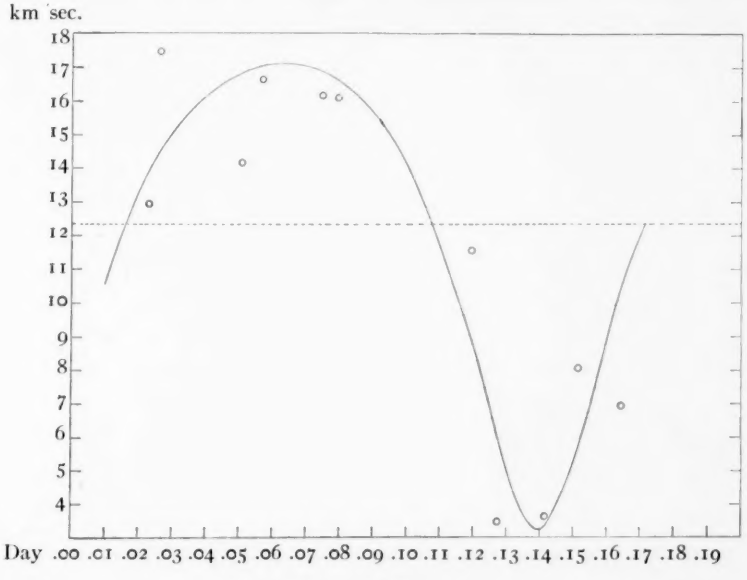

FIG. 2

than in the second series. The plates are fairly uniform in density and were used in an investigation made to observe variations in the intensities of the various lines in the spectra. The period used in the 1931 study, $0^d.1556$, seemed to satisfy the observations. While a variation in the elements was observed, the change in K and ω was not great. However, the change in γ and e seems real enough to conclude that there is a variation of these quantities. The curve for this series is given in Figure 3.

It will be noted that there is an indication of a depression at the maximum velocity in Figure 3. This seems to be real. There is a slight indication of such a depression in the observed velocities of the second series. It is also worthy of note that just before the depres-

sion the two *O* II lines, λ 4072 and λ 4075, are good, but that at this point they begin to grow weak and after the observed depression they practically disappear. The lines λ 4414, *O* II and λ 4416 seem to be weak at both maximum and minimum velocity and also on the descending branch of the curve, while on the ascending branch they

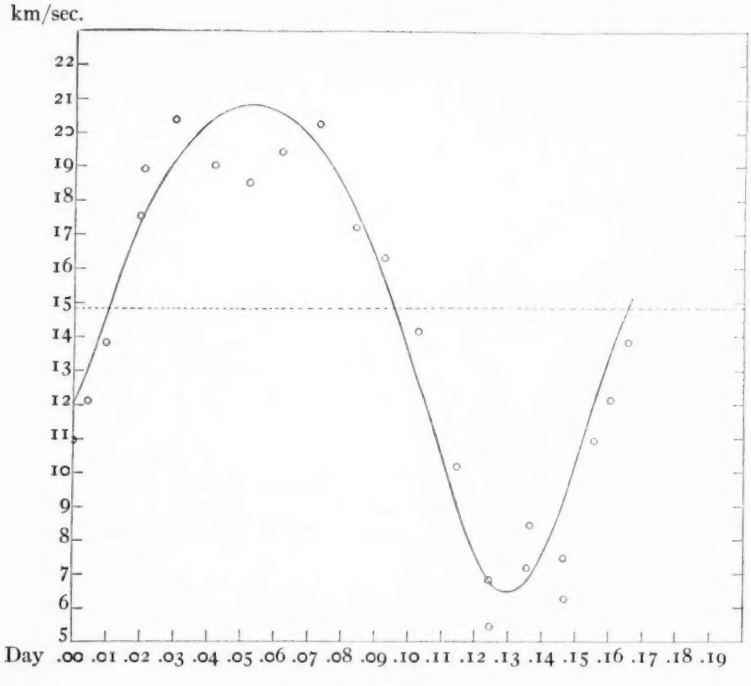

FIG. 3

are uniformly stronger than in other parts. It seems safe to say from a study of the lines in this series that at least some of the lines in the spectra vary in intensity and that there is probably a phase relation in this variation, but it would be unwise to base so important a conclusion on one series of spectrograms. Unfortunately, the spectra of the first two series considered here are not adequate for such a study.

YERKES OBSERVATORY
August 26, 1933

A STATISTICAL STUDY OF ROTATIONAL BROADENING IN 112 STARS OF CLASS F

By CHRISTINE WESTGATE

ABSTRACT

The width of the spectral line λ 4215 is recorded for 112 stars of class F. A diagram illustrating the frequency of occurrence of different widths among the early and among the late F's shows that appreciable rotation disappears in the middle F's. The frequency diagram for class F is compared with those for classes B and A as determined in previous papers. The indication is that stars of class A have greater velocities of rotation than those of either class B or class F. There is no definite relation between absolute magnitude and rotational velocity. The average observed rotational velocity is independent of galactic latitude, which indicates that the axes of the stars are directed at random.

1. The rotational broadening of spectral lines in stars of classes B and A has been investigated in two earlier papers.[1] This work is now extended to stars of spectral class F. Table I contains a list of 112 F-type stars for which the width of the *Sr* II line at λ 4215 was measured with a comparator. For 69 of the stars two spectrograms of each were measured; for the remaining 43, only one spectrogram of each was available. Those stars for which only one spectrogram was measured are indicated in the second column of Table I by a (1) after the Δλ, which is expressed in angstrom units. The data were obtained from one-prism spectrograms taken with the Bruce spectrograph attached to the Yerkes 40-inch refractor. The dispersion at λ 4500 is 30 A/mm.

The absorption line of ionized strontium at λ 4215.5 may be at times blended with a weak line of iron. This line was chosen for the study after an examination of good plates of narrow-line stars of all subdivisions of class F and of very different luminosities had made it evident that λ 4215 is probably the most suitable line for this work. Any broadening of this line could henceforth be regarded as due largely to a rotational Doppler effect.

2. In this paper the frequencies will be expressed as percentages of the total number of stars considered in each group. The widths of λ 4215 have not been transformed into observed velocities of rota-

[1] *Astrophysical Journal*, **77**, 141, and **78**, 46, 1933.

tion because an adequate means of conversion is lacking. Therefore, in comparing the rotations of stars of classes B, A, and F, the broadening of the respective spectral lines has been used. The fre-

TABLE I

Boss	Width λ 4215 in A	Boss	Width λ 4215 in A	Boss	Width λ 4215 in A
12	1.4 (1)	2853	1.7 (1)	4483	1.6 (1)
19	1.6 (1)	2999	1.3	4504	1.3 (1)
168	1.2	3063	1.6	4528	1.2 (1)
189	1.6	3224	1.6 (1)	4539	1.4 (1)
325	1.2	3307 (S.)	1.3	4542	1.4
421	2.4	3307 (N.)	1.4	4559	1.3
452	1.2	3371	1.5	4672	1.4 (1)
617	1.3 (1)	3412	1.6	4707	1.5 (1)
629	1.9	3415	2.3 (1)	4731	1.3 (1)
644	3.8 (1)	3422	1.6 (1)	4753	1.3
654	1.5	3447	1.3	4820	1.6
772	1.3 (1)	3518	3.5 (1)	4911	1.8 (1)
847	1.4 (1)	3526	3.2 (1)	4924	1.2 (1)
924	1.0 (1)	3558	1.2	4953	2.1
935	1.5	3635	1.7 (1)	4962	1.4 (1)
971	1.4 (1)	3679	1.6	5000	1.6
1007	2.8	3704	1.5	5014	1.3
1027	2.3	3722	3.2	5028	4.0 (1)
1046	1.8	3729	0.7 (1)	5213	1.6 (1)
1051	1.9 (1)	3784	1.5	5229	1.2
1056	2.4	3855	1.7	5255	1.1
1083	1.7 (1)	3926	2.3	5291	1.6
1092	1.6	3940	1.6	5443	1.7
1140	1.4 (1)	3960	2.3	5455	1.3 (1)
1143	2.6	4055	1.6	5460	2.5
1153	2.0	4057	1.9	5461	1.4
1187	1.2	4077	1.4 (1)	5555	4.5 (1)
1424	1.6 (1)	4082	1.5	5562	1.3
1839	1.3	4090	1.1 (1)	5587, 8	1.3
2008	1.1	4163	2.0	5688	1.3
2058	1.7	4165	3.4	5742	2.4
2354	1.7	4181	0.9 (1)	5838	2.6
2413	1.7	4206	1.5	5874	0.8 (1)
2552	1.4	4270	1.6 (1)	5975	1.6
2602	1.5	4300	1.7	6077	1.4 (1)
2632	2.7	4423	1.5	6135	2.1 (1)
2730	2.4	4470	1.7	6156	1.3 (1)
2935	1.8				

quency-curves are shown in Figure 1*a*. The frequency-curve obtained from λ 4472 for the stars of type B is not repeated here, since the use of *He* I is open to objection because of its susceptibility to Stark broadening. The effect of spectral class upon the width of a line must be considered in a comparison of speeds of rotation of stars

in different classes. Since the sharpest line which was measured in class A is 0.4 A wider than any line measured in a star of class B or class F, that amount has been subtracted from the measurements of widths in the former class. No correction has been applied to the values of $\Delta\lambda$, in order to allow for the difference in wave-length between λ 4481, used for the B and A stars, and λ 4215, used for the F stars.

Figure 1*a* indicates that the stars of class A tend to have greater velocities of rotation than those of either class B or class F.[2] Atten-

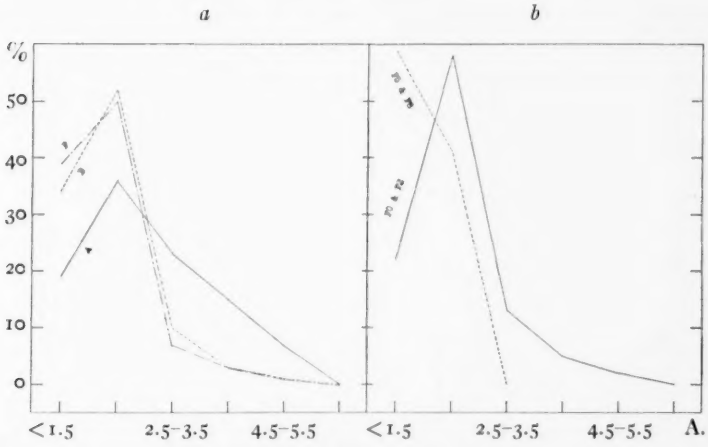

FIG. 1.—Frequency diagrams. Abscissae are widths of λ 4215 in angstrom units. Ordinates are percentages of the total number of stars in the respective classes.

tion is called to the fact that the figures give the observed quantities and not the actual rotational velocities. The latter could be obtained by a statistical correction for inclination.[3]

3. The subdivisions of classes A and B show no variation in the form of the frequency diagrams. However, the observed widths of λ 4215 show quite a different frequency distribution in the early F's than in the late F's. The dotted line in Figure 1*b* is the frequency-curve for 51 stars of classes F5 and F8, while the continuous line is for 61 stars of classes F0 and F2. Appreciable rotation seems to dis-

[2] O. Struve and C. T. Elvey, *Monthly Notices of the Royal Astronomical Society*, **91**, 673, 1931.

[3] Westgate, *loc. cit.*

appear rather suddenly between F2 and F5. It is surprising that there is so much similarity in the frequency-curves for classes B, A, and early F. The limited amount of material available at the Yerkes Observatory for class G and later classes shows that in the cooler stars the rotational velocities are small.

4. The data were examined for evidences of a dependence of line widths upon absolute magnitudes in stars of classes A and F. Absolute magnitudes for 202 stars of class A and for 82 stars of class F were obtained from the trigonometric parallaxes listed in "Catalogo generale di parallasi stellari" by Gino Cecchini.[4] The observations indicate that there is no certain effect of absolute magnitude upon the widths of the spectral lines considered here, but the material on trigonometric parallaxes is not sufficient for a detailed study.

5. A study has been made of the relation of the broadening of spectral lines to the galactic latitudes of the stars concerned. None of the subdivisions of classes A and F show any indication of a relation of the magnitude of stellar rotation to galactic latitude. This result justifies the assumption originally made[5] that the axes of the stars are directed at random. The B-type stars are so concentrated toward the plane of the Milky Way that it was not possible to investigate the average rotational velocities for different galactic latitudes.

YERKES OBSERVATORY
November 1933

[4] *Pubblicazioni del R. Osservatorio Astronomico di Merate (Como)*, No. 4, 1931.

[5] *Astrophysical Journal*, **77**, 150, 1933.

A SOLUTION OF THE LIGHT-CURVE OF X TRIANGULI

BD $+27^{\circ}318$, $\alpha = 1^{h}54^{m}.9$, $\delta = +27^{\circ}24'$ (1900)

By VIRGINIA MODESITT and TED MARTIN

ABSTRACT

The solution is based on Jordan's photographic observations. The light-curve shows both ellipticity and reflection effects, a total primary, and uniform brightness of the disks.

The observations used in the determination of this light-curve are those of Professor Jordan,[1] made at the Allegheny Observatory with the 30-inch Thaw refractor. They cover the entire light-curve.

The light-elements used in this solution are given in the formula

$$\text{Min.} = \text{J.D. } 2423130.3136 + 0^{d}.971534E .$$

The initial epoch is due to Leiner and the period to Dugan. Both are used by Dugan in his visual curve.[2] Jordan used the same initial epoch and a period of $0^{d}.9715344$. The change in the period gives the tabulated small corrections to his published phases below. A plot of the observations in primary minimum for nights 1–15 indicated that a further correction of +0.001 to all phases was necessary.

Night	Correction	Night	Correction
2	+0.001	13	−0.001
5	.001	15	+ .001
6	.001	20	+0.001
10	+0.001		

Professor Jordan used sets of four plates, A, B, C, and D, in his observing program. A letter from him, listing the plates, omitted made it possible to associate each observation with a particular plate. Hence in the plots of the observations, night by night, single plates which were not in accordance with the rest in the set could be adjusted. These adjustments are as indicated in the table. The observations on night 8 were not quite as consistent as those on other

[1] *Allegheny Obs. Pub.*, **7**, 171, 1929.

[2] *Contr. from the Princeton U. Obs.*, No. 8.

nights, even with the adjustments made, and it seemed best to give Nos. 167–173 zero weight in the solution.

Night	Plate	Δm
6	B	−0.02
8	A	− .07
12	B	− .04
14	A, B	− .06
15	A	− .07
19	D	+0.04

As would be expected, the range of magnitude, $3^{m}02$, is somewhat greater for the photographic than for the visual observations. No shift of secondary from mid-period is evident. Both ellipticity and

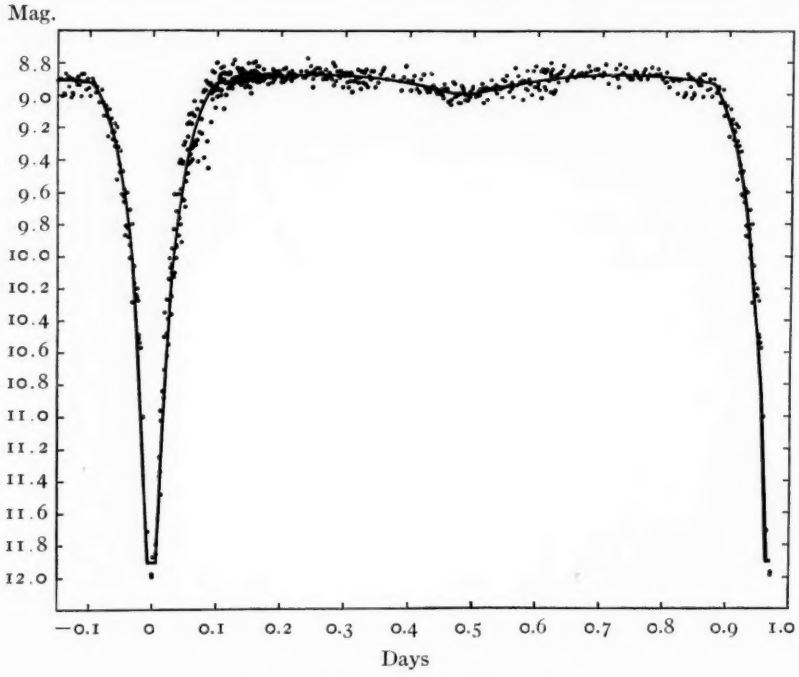

FIG. 1.—Light-curve of X Trianguli

reflection effects are found, though the latter is small; a least-squares solution was used in the determination of the constants defining them. Since the loss of light in the system at primary and at

secondary is greater than unity, a solution would not be physically possible without these quantities.

$$\text{Maximum} = 8^{m}.864$$
$$z = 0.160$$
$$s = 0.0070$$

The solution still presented difficulties. A total eclipse is indicated at primary by the slope of the shoulders of the curve. The relatively wide spread of the observations at secondary and the rapidity of the light-change at primary made it difficult to be sure of the exact depth. Finally, however, from a plot of intensity against sin θ, the bottom of the primary was taken at 0.067 and that of the secondary at 0.955. This limited the solution sharply to a k of 0.82. The final curve with A = 0.054 and B = 0.044 gave so satisfactory a fit that

SUMMARY OF RESULTS

R.A. (1900)	$1^{h}54^{m}.9$
Decl. (1900)	$+27°24'.0$
Maximum	$8^{m}.864$
Primary min	$11^{m}.886$
Secondary min	$8^{m}.986$
Epoch	2423130.3136
Period	$0^{d}.971534$
	Uniform
Nature of eclipse	Total primary
Length of eclipse	$0^{d}.198$
Phase of secondary	$\frac{1}{2}P$
$e \cos \omega$	0
Inclination	87°.1
$1-\lambda_1$	0.933
$1-\lambda_2$	.045
k	.82
b/a	.916
c/a	.873
L_2	.933
L_1	.067
L_1-2s	0.053
J_2/J_1	20.7
r_2	0.2631
r_1	0.3208

it seemed doubtful if much could be gained in going to a darkened solution. The elements, allowing for the differences between the photographic and visual observations, are in good agreement with those of Dugan except for the matter of darkening.

The magnitudes plotted on Figure 1 are those of Professor Jordan as given in his published list. The phases used are Professor Jordan's, adjusted by the amounts noted in this paper.

We are indebted to Dr. J. E. Merrill for his suggestions in the solution of this problem. Miss Laura Ewalt helped in the preliminary solution.

URBANA, ILLINOIS
November 29, 1933

NOTES

MEASURES OF THE EXTENT OF THE CHROMOSPHERE

ABSTRACT

The height of the chromosphere in *Hα*, *Hβ*, and D_3 is determined from spectra taken with a radial slit. The results are in agreement with visual observations in giving values about one-half of those derived from flash spectra.

The measures summarized herein were made on spectral line extensions of the solar limb photographed at the McMath-Hulbert Observatory during the summer of 1933. The spectra were secured with the spectroheliokinematograph[1] of that Observatory by the simple expedient of placing the Anderson prisms with their faces parallel to the plane of the slits and opening the second slit so that a portion of the solar spectrum could be photographed upon the motion-picture film. It is well known that when the slit of a spectroscope is placed radial to the solar limb, the more prominent chromospheric lines may be observed as bright extensions, the lengths of which will give a measure of the height of the chromosphere. The writer was able to see and photograph the lines *Hα*, *Hβ*, and D_3, from which chromospheric extents were determined.

The McMath-Hulbert telescope proved to be excellent for this work, the methods developed there for celestial kinematography enabling one to drive the telescope at the solar rates in both co-ordinates while photography is in progress. Observations were made with three different mirror combinations of focal lengths 210, 528, and 830 inches, respectively. The 528-inch combination was finally adopted as giving the best results; all the values tabulated being determined with that focal length.

Exposures were made on four different dates during the summer, at position angles on the solar limb of 45°, 90°, and 135°. Eastman supersensitive panchromatic film, double coated and with a gray

[1] *Publications of the Observatory of the University of Michigan*, **5**, 103, 1933.

base, was used. This emulsion has rather low contrast, but is particularly free from halation.

The values tabulated below are based upon several sets of exposures, the number of observations going into the mean values being 40 for $H\alpha$, 21 for $H\beta$, and 7 for D_3. The probable errors are derived from the internal agreement between the various days. For comparison, the results of other observers are listed in Table I.

TABLE I

CHROMOSPHERE HEIGHTS

Spectral Line	Measured Height	Other Observers		
		Abetti	Fox	Pettit
$H\alpha$	5600 ± 190 km	6800 km	6900 km	5500 km
$H\beta$	4100* ± 170	.....	.....	.....
D_3	4700 ± 270	.....	4700	.....

* Keenan (*Astrophysical Journal*, **75**, 277, 1932), in the photometry of $H\beta$, was able to trace the chromospheric line to 4000 km.

Abetti's[2] results are from visual measures with a spectroscope; Fox[3] used a visual method of determining the marginal appearance of the chromospheric lines; Pettit's[4] results are from visual measures of the diameter of the solar disk, with the spectrohelioscope, in the light of $H\alpha$ and of the continuous spectrum. The photographic method agrees with the visual measures, although the chromospheric extent, determined without an eclipse, is only about one-half the values derived by Mitchell[5] from eclipse flash spectra.

These observations are not sufficiently extensive to distinguish the variations of chromospheric height with solar latitude.[6] The writer plans to secure more extensive data by the photographic method during the coming summer for the further examination of height variations.

[2] *Publications of the Arcetri Observatory*, 1922 ff.

[3] *Astrophysical Journal*, **57**, 234, 1923.

[4] *Mount Wilson Contributions*, No. 451, p. 13, 1932.

[5] *Astrophysical Journal*, **71**, 1, 1930.

[6] *Handbuch der Astrophysik*, **4**, 139, 1929.

In conclusion, I wish to extend my thanks to Mr. R. R. McMath, who enabled me to secure the observations, and to Professor Keivin Burns for his interest and advice.

R. M. PETRIE

THE OBSERVATORY,
UNIVERSITY OF MICHIGAN,
January 11, 1934

REVIEWS

The New Background of Science. By SIR JAMES JEANS. New York: Macmillan Co., 1933. Pp. 301. 1 pl. $2.50.

In the flood of surveys and interpretations of modern physical science by the sheer bulk of which the groping reader is almost washed away, the latest contribution of Sir James Jeans will receive attention not because of any great originality of concept but rather for the charm of its presentation. There is little in it that has not been said before, by Jeans or by others, but few are the writers who can attain at once to the brilliance and the simplicity with which Jeans pictures the most recondite subjects.

Beginning with a short history of scientific thought, he devotes the bulk of his book to an exposition of the fundamental ideas of relativity, modern quantum theory, and wave mechanics. Throughout the discussion the old distinction between objective nature and the subjective world of our sense impressions is amplified and stressed. The elements of our pictures of the phenomena of nature are necessarily concepts with which our minds are familiar, but the elements of reality need not be so.

> . . . . It would seem that science might legitimately proceed along the road from phenomena to reality by thinking over unfamiliar concepts until they become familiar, the concepts being selected in the first instance on grounds of probability, as appearing likely to figure in ultimate reality.

From this point of view our picture of matter and radiation as composed of distinct particles—protons, electrons, and photons—having familiar mechanical properties appears as only a first approximation. The fact that many phenomena can be predicted only by assigning to matter the characteristics of waves has forced us to revise the picture. We think next of our particles as having their positions described approximately in terms of waves of probability. However, even now the wave concept and the particle concept are not entirely compatible. Such

experiments as those of Stern and Gerlach on the behavior of streams of atoms in magnetic fields suggest that we must abandon entirely the particle picture, and with it the possibility of representing matter in a framework of space and time. Such a readjustment of our attitude toward nature is exceedingly difficult, but if a purely mathematical representation proves more accurate than any concrete model in describing the experiences which are our only direct means of contact with nature, we have no recourse but to accept space, time, and matter as mathematical concepts.

This picture is not final; it marks merely the present state of our knowledge. "We must admit it as conceivable that the further advances of science may yet clothe our present mathematical abstractions in new dresses of physical reality, and possibly even of material substance."

However, nothing can be predicted with certainty, and in emphasizing this point Jeans's writing takes on a stronger metaphysical tone. Indeterminacy is his theme and uncertainty his thesis. Heisenberg's principle of indeterminacy in atomic physics follows from the fact that at best our means of studying nature suffer from a coarse-grainedness which sets a final limit to the accuracy of our data. In addition to this subjective uncertainty there may well be a lack of determinism in the world of reality. At the moment science favors the viewpoint of the idealist while the realist with his notions of strict causality stands out in the cold, but it is possible that the situation may be reversed in time to come.

P. C. KEENAN

Astronomy, an Introduction. By ROBERT H. BAKER. 2d ed. New York: D. van Nostrand Co., 1933. Pp. xix+522. $3.75.

This excellent textbook was originally published in 1930, and has twice been reprinted in three years. The second edition contains many new features, such as accounts of two asteroids which approach the earth closer than Eros, of the globular clusters around the Andromeda nebula, of the red shift of spiral nebulae, and of the theory of the expanding universe. Several new illustrations have been added, but the total number of pages has not been increased. The high standard of the first edition has been fully maintained.

O. S.